# Gauge-Higgs Unification with Kinetic Brane Terms

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# Outline

- Introduction (EWSB and SM parameters),
- Phenomenological EW-Higgs Unification[a],
- 6D Gauge-Higgs unification with $SU(3)_W$,
- Brane Kinetic Terms and $\theta_W$,[b],
- Higgs mass with Brane Kinetic Terms,
- Conclusions,

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[a] Based on work donde with A. Aranda and A. Rosado, MPL(2006)

[b] Based on work donde with A. Aranda, PLB(2006)

# Introduction

- EWSB needed to generate SM gauge and fermion masses.
- SM Higgs doublet $\Phi(x)$ has the potential: $V(\Phi) = -\mu^2|\Phi|^2 + \lambda|\Phi|^4$
- One unknown parameter $\lambda$ determines $M_H$
- Exp. and precision analysis seems to prefer a "light" Higgs boson, i.e. $116 \leq m_h \leq 180$ GeV.
- LHC is expected to detect at least one Higgs boson, while its precise nature will be tested at LC.

# Hierarchy Problems

- Large Hierarchy: a severe fine tuning problem,
  Scalars get quad. rad. corrs.: $\delta M_H^2 \sim C_i \Lambda^2$
  A mechanism should protect Higgs mass
  when $\Lambda >> m_W$ , i.e. $C_i \to 0$ ,
- Little Hierarchy
  Higgs mass should not be far from $\Lambda_{eff}$,
  Global fits suggest $\Lambda_{eff} \sim 400$ GeV,
  EW precision measurements imply,
  $\Lambda_{eff} \sim 5 - 10$ TeV

# Models of EWSB

- Traditional solutions to EWSB problems:
  - Weak/perturbative theories, e.g. SUSY,
  - Strongly-interacting dynamics, e.g. TC
- New approaches proposed recently are based on:
  New Dynamics: - Little/Fat Higgs,
  - String motivated.....
  Extra Dimensions: - EWSB by O.B.C.
  -Higgsless EWSB, - Warped-AdS/CFT,
  - Gauge-Higgs unification

# SM Parameters

- Gauge parameters (dimensionless)
  - Gauge couplings: $g_3, g_2, g_Y$
  - Strong phase: $\theta_{QCD}$
- Higgs (dimensionfull) parameter: $\mu^2$
- Higgs self-coupling: $\lambda$
- Yukawa Couplings $Y_{ij}^f$
- Could it be possible to express all SM parameters in terms of gauge parameters?

# Phenomenological EW-Higgs Unification

- Higgs self-coupling should be related to EW gauge couplings,
- This relation could take a generic form: $\lambda = f(g_1, g_2)$
- To simplify we mposse a linear (I) or quadractic (II) relation at a high scale,
- The quadractic relation (case II) takes the form: $g_i^2 = k_H \lambda$,
- The normalization factor $k_H$ is taken as: $0.1 < k_H < 10$

# Phenomenological EW-Higgs Unification

- What is the high scale? We take it where $g_1 = g_2$,
- Result depends on hypercharge normalization, i.e. $g_1^2 = k_Y g_Y^2$,
- We considered $k_Y = \frac{5}{3}, \frac{3}{2}, \frac{7}{4}$,
- Use RGE to get $\lambda$ at EW scale and determine the Higgs mass.
- We obtain the result: $m_h \simeq 180$ GeV.

# XD and Gauge-Higgs Unification

- Basic idea: Identify the Higgs as a component of a higher dimensional gauge field ($A_{\hat{\mu}}$)

$$A_{\hat{\mu}} \rightarrow \begin{array}{l} A_\mu = \text{4D gauge bosons} \\ A_M = \text{4D scalars} \end{array}$$

- **5D Models:**
  Generally predict a very small Higgs mass
- However, in **6D Models**:
  Quartic coupling present at tree level
  Higgs mass prediction is better
  But EWSB stability must be verified

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- $F_{\hat{\mu}\hat{\nu}} = \partial_{\hat{\mu}} A_{\hat{\nu}} - \partial_{\hat{\nu}} A_{\hat{\mu}} - i g_6 [A_{\hat{\mu}}, A_{\hat{\nu}}]$

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- $F_{\hat{\mu}\hat{\nu}} = \partial_{\hat{\mu}}A_{\hat{\nu}} - \partial_{\hat{\nu}}A_{\hat{\mu}} - ig_6[A_{\hat{\mu}}, A_{\hat{\nu}}]$
- Spacetime (for $N = 2$): $A^\mu \to A^\mu$, $A^M \to -A^M$, $M = 5, 6$.
- Gauge: for the generators of SU(3), $t_a \to \Theta^{-1}t_a\Theta$, where

$$\Theta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} .$$

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- $A_\mu = \sum_{a=1,2,3,8} A_\mu^{(a)} \frac{\lambda_a}{2}$
- $H_M = \sum_{a=4,5,6,7} A_M^{(a)} \frac{\lambda_a}{2}\ ;\ \ M = 5\,,6$

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$$A_\mu = \frac{1}{2}\begin{pmatrix} A_\mu^{(3)} + \frac{1}{\sqrt{3}}A_\mu^{(8)} & A_\mu^{(1)} - iA_\mu^{(2)} & 0 \\ A_\mu^{(1)} + iA_\mu^{(2)} & -A_\mu^{(3)} + \frac{1}{\sqrt{3}}A_\mu^{(8)} & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}}A_\mu^{(8)} \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} W_\mu^{(3)} + \frac{1}{\sqrt{3}}A_\mu^{(8)} & \sqrt{2}W_\mu^{+} & 0 \\ \sqrt{2}W_\mu^{-} & -W_\mu^{(3)} + \frac{1}{\sqrt{3}}A_\mu^{(8)} & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}}A_\mu^{(8)} \end{pmatrix}$$

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$$H_M = \frac{1}{2}\begin{pmatrix} 0 & 0 & A_M^{(4)} + iA_M^{(5)} \\ 0 & 0 & A_M^{(6)} + iA_M^{(7)} \\ A_M^{(4)} - iA_M^{(5)} & A_M^{(6)} - iA_M^{(7)} & 0 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & H_M^{*+} \\ 0 & 0 & H_M^{0} \\ H_M^{-} & H_M^{0*} & 0 \end{pmatrix}$$

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$$\begin{aligned}\mathcal{L}_4^{(0)} &= -\frac{1}{4}(F^a_{\mu\nu})^2 - \frac{1}{4}(B_{\mu\nu})^2 \\ &+ \left|\left(\partial_\mu - ig_4 W^a_\mu \frac{\tau^a}{2} - ig_4 \tan\theta_W \frac{1}{2} B_\mu\right)\mathcal{H}\right|^2 - \frac{g_4^2}{2}|\mathcal{H}|^4\end{aligned}$$

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- $\mathcal{H} = \begin{pmatrix} H_M^{*+} \\ H_M^0 \end{pmatrix}$
- $\tan\theta_W = \sqrt{3}$
- $V_{class}(\mathcal{H}) = \frac{g_4^2}{2}|\mathcal{H}|^4$

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Approximately

$$V_{quant}(\mathcal{H}) = -\mu^2|\mathcal{H}|^2 + \lambda|\mathcal{H}|^4$$

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- Use $\langle|\mathcal{H}|\rangle = v/\sqrt{2}$
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$$\rightarrow \frac{m_H}{m_W} = \frac{2\sqrt{2\lambda}}{g} = 2$$ [a]

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[a] C.A. Scrucca, M. Serone, L. Silvestrini, A. Wulzer, JHEP **0402** (2004) 049; A. Wulzer, hep-th/0405168; C. Biggio, M. Quirós; hep-ph/0407348

# 6D SU(3) with Brane Kinetic Terms (BKT)

In order to fix the value of $\tan\theta_W$ we introduce brane kinetic terms by considering the following possibilities:

- **A:** $\mathcal{L}_{TCB} = -\frac{1}{4}\, c\, \delta(x_5)\, \delta(x_6)\, F^{\mu\nu} F_{\mu\nu}$

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- **B:** $\mathcal{L}_{TCB} = -\frac{1}{4}\, (c_5\, \delta(x_5)\, F^{mn} F_{mn} + c_6\, \delta(x_6)\, F^{st} F_{st}),$
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  $m, n = 0, 1, 2, 3, 6;\ s, t = 0, 1, 2, 3, 5$
- **C:** $\mathcal{L}_{TCB} = -\frac{1}{4}\delta(x_5)\delta(x_6)\left[c_1\left(F^{(a)}_{\mu\nu}\right)^2 + c_2\left(F^{(8)}_{\mu\nu}\right)^2\right]$
  $a = 1, 2, 3$

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- $\mathcal{Z}_{1,2} = 1 + \frac{c_{1,2}}{\pi^2 R_5 R_6}$
- Study parameter space consistent with $\tan\theta_W$
- Check the Higgs mass

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**Weinberg Angle**: Defining $g = g_4/\sqrt{\mathcal{Z}_1}$ and $g' = \sqrt{3}g_4/\sqrt{\mathcal{Z}_2}$

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**Higgs Mass**: $\frac{m_H}{m_W} = 2\sqrt{\mathcal{Z}_1}$

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