F-TERM HYBRID INFLATION Followed by Modular Inflation

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OUTLINE:

- MOTIVATION
- MODELS OF F-TERM HYBRID INFLATION (FHI)
- THE BASICS OF MODULAR INFLATION (MI)
- OBSERVATIONAL CONSTRAINTS
- NUMERICAL RESULTS
- CONCLUSIONS

0. INTRODUCTION TO INFLATION





We have $\ln |\Omega - 1| \propto 6\tau [4\tau]$ for RD [MD] but $\ln |\Omega - 1| \propto -2\tau$ for VD. Note that $H \propto R^{-2} [R^{-3/2}]$ for RD [MD] but H =cst for VD.

A Lenght $\lambda = 2\pi R/k$ is Inside [Outside] the Horizon 1/ when $\lambda < 1/H \Leftrightarrow k > RH$ [$\lambda > 1/H \Leftrightarrow k < RH$]. In order to Resolve the Horizon Problem we Need $d(\lambda H)/dt > 0 \Leftrightarrow \ddot{R} > 0$ (note that $\lambda \propto R$).

B. INFLATION AND THE INFLATON

FROM THE EQUATIONS OF THE COSMOLOGICAL EVOLUTION, WE DEDUCE THAT ACCELERATION OF TH UNIVERSE MEANS:

$$\frac{\ddot{R}}{R} = (1-\epsilon)H^2 = -\frac{1}{6m_{\rm P}}(1+3w)\rho > 0 \quad \Leftrightarrow \quad \left\{ \begin{array}{ll} \epsilon < 1 & \text{where} \quad \epsilon = -\dot{H}/H^2, \\ w < -1/3 & \text{where} \quad w = P/\rho. \end{array} \right.$$

Since the Density ρ_{ϕ} and the Pressure P_{ϕ} of a omogenous Scalar Field $\phi(t)$ are:

$$\rho_{\phi} = \frac{1}{2}\dot{\phi}^2 + V(\phi) \quad \text{and} \quad P_{\phi} = \frac{1}{2}\dot{\phi}^2 - V(\phi) \quad \text{We Can Obtain} \quad P_{\phi} = -\rho_{\phi} \quad (w = -1) \quad \text{if} \quad \dot{\phi} \ll V(\phi).$$

or $H^2 = \rho_{\phi}/3m_{\rm P}^2 = {\rm cst.}$ As a Consequence $R(t) = R_{\rm i}e^{\Delta N_e}$ while $T(t) = T_{\rm i}e^{-\Delta N_e}$ Where $\Delta N_e = H(t - t_{\rm i})$ is the Number of e-Foldings During Inflation for $t > t_{\rm i}$.

If in Addition we have $\dot{\phi} \gg \ddot{\phi} \Leftrightarrow \eta < 1$, Where $\eta = m_{\rm P}^2 d^2 V(\phi)/d\phi^2/V$ We Obtain "Slow Roll" Inflation and the Equation of Motion (: EOM) of ϕ :

$$\ddot{\phi} + 3H\dot{\phi} + dV(\phi)/d\phi = 0$$
 Can be Written as $-3H\dot{\phi} = dV(\phi)/d\phi$.

C. PRIMORDIAL CURVATURE PERTURBATIONS



Expanding in Fourier Series the Perturbations $\delta\phi(\mathbf{x},t)$ of ϕ , $\delta\phi(\mathbf{x},t) = \phi(\mathbf{x},t) - \phi(t)$,

$$\delta\phi(\mathbf{x},t) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} e^{i\mathbf{k}\mathbf{x}} \,\delta\phi_{\mathbf{k}}(t),$$

We Obtain a Power Spectrum of the mode $\delta\phi_{\mathbf{k}}$, $P_{\phi}^{1/2} = (k^3/2\pi)^{1/2}\delta\phi_{\mathbf{k}}(k=RH)$ which Results to a Power Spectrum of Curvature Perturbations $P_{\mathcal{R}}^{1/2} = H^2/\dot{\phi}P_{\phi}^{1/2}$. This is Related to the Quadripole Anisotropy $\Delta T/T$ of CMB Measured by WMAP.

I. MOTIVATION

A. WMAP3 AND FHI

Fitting the WMAP3 Data With the Standard Power-Law Cosmological Model Λ CDM, One Obtains¹ That, at the Pivot Scale $k_* = 0.002/Mpc$,

 $n_{
m s} = 0.958 \pm 0.016 \ \Rightarrow \ 0.926 \lesssim n_{
m s} \lesssim 0.99$ (95% c.l.)

These Results Bring Under Considerable Stress a Class of SUSY Models of FHI, Realized at (or Very Close to) $M_{\rm GUT} = 2.86 \times 10^{16} \text{ GeV}$ which Predicts (For $N_{\rm tot} = N_{\rm HI*} = 50$)²:

 $n_{\rm s}\sim 0.98$ or even $n_{\rm s}\sim 1,$

IF SUGRA CORRECTIONS (WITH CANONINAL KÄHLER POTENTIAL) ARE INCLUDED³.

B. PROPOSED SOLUTIONS

- Utilization of a quasi-Canoninal Kähler Potential Which Can Generate a Maximum on the Inflationary Path (Hilltop Inflation). Indispensable Tuning of the Initial Conditions (~ 0.01) is Needed⁴.
- Inclusion of a Small Contribution to $P_{\mathcal{R}}$ from Cosmic Strings, Which, However, Requires $M \ll M_{\text{GUT}}^5$.

C. COMPLEMENTARY INFLATION

Our Proposal is Based on the Observation That $n_{\rm S}$ Within FHI Generally Decreases With the Number of e-Foldings, $N_{\rm HI*}$, That k_* Suffered During FHI: e.g., for st-FHI with Superpotential $W = \kappa S (\bar{\Phi} \Phi - M^2)$ (see Below):

M.U. Rehman, V.N. Şenoğuz, and Q. Shafi (2006).

¹D.N. Spergel et al. (2006)

²G.R. Dvali, Q. Shafi, and R.K. Schaefer (1994); G. Lazarides, R.K. Schaefer, and Q. Shafi (1997).

³V.N. Şenoğuz and Q. Shafi (2003).

⁴L. Boubekeur and D. Lyth (2005); M. Bastero-Gil, S.F. King, and Q. Shafi (2006); B. Garbrecht, C. P., and A. Pilaftsis (2006);

⁵R.A. Battye, B. Garbrecht, and A. Moss (2006); R. Jeannerot and M. Postma (2005); J. Rocher and M. Sakellariadou (2005).



We Observe that For Relatively Large $\kappa~(\simeq~0.01,0.1)$ and $N_{\rm HI*}\sim~(15-20)$ we can Obtain $n_{\rm s}\simeq~0.96$. The Residual Number of e-Foldings $N_{\rm tot}-N_{\rm HI*}$ (Required for the Resolution of the Horizon and Flatness Problems Of SBB) Can be Obtained by A Second Stage Of Inflation Realized at a Lower Scale.We Call this type Of Inflation <u>Complementary</u> Inflation. We can Show that MI Can Naturally Play This Role.

II. MODELS OF FHI

A. THE RELEVANT SUPERPOTENTIAL

THE FHI CAN BE REALIZED ADOPTING ONE OF THE SUPERPOTENTIALS⁶⁷⁸:

$$W = \begin{cases} \kappa S \left(\bar{\Phi} \Phi - M^2 \right) & \text{for standard FHI (: st-FHI),} \\ \kappa S \left(\bar{\Phi} \Phi - M^2 \right) - S \frac{(\bar{\Phi} \Phi)^2}{M_S^2} & \text{for shifted FHI (: sh-FHI),} \\ S \left(\frac{(\bar{\Phi} \Phi)^2}{M_S^2} - \mu_S^2 \right) & \text{for smooth FHI (: sm-FHI),} \end{cases}$$
 Where:

⁶sm-FHI: G.R. Dvali, Q. Shafi, and R.K. Schaefer (1994)

⁷sh-FHI: *R. Jeannerot, S. Khalil, G. Lazarides and Q. Shafi (2000)*

⁸sm-FHI: G. Lazarides and C. Panagiotakopoulos (1995); R. Jeannerot, S. Khalil, and G. Lazarides (2001)

- $\overline{\Phi}$ and Φ : Pair of Left Handed Superfields Belonging to Non-Trivial Conjugate Reps of a Gauge Group G and Reducing its Rank by Their vevs (Waterfall Fields),
- $M_{
 m S}\sim 5 imes 10^{17}\,{
 m GeV}$: The String Scale,
- κ and M, $\mu_{\rm S}$ (~ $M_{\rm GUT}$): Positive Parameters.

$$W \ni \begin{cases} \text{Renormalizable Terms Consistent With} \\ U(1)_R : S \to e^{i\alpha} S, \ \bar{\Phi}\Phi \to \bar{\Phi}\Phi, \ W \to e^{i\alpha}W & \text{for st-FHI}, \\ \text{Leading Non-Renormalizable Term} & \text{for sh-FHI}, \\ Z_2 & \text{Invariant Terms Under Which } \Phi \to -\Phi & \text{for sm-FHI}. \end{cases}$$

B. THE SUSY POTENTIAL

1. DERIVATION

The SUSY Potential Includes 2 Contributions: $V_{\rm SUSY} = V_{\rm F} + V_{\rm D},~$ Where

• D-Term Contribution:
$$V_{\rm D}=0,~~{\rm With}~~|\bar\Phi|=|\Phi|.$$

• F-Term Contribution:
$$V_{\rm F} = \begin{cases} \kappa^2 M^4 \left((\bar{\Phi}^2 - 1)^2 + 2\bar{S}^2 \bar{\Phi}^2 \right) & \text{for st-FHI}, \\ \kappa^2 M^4 \left((\bar{\Phi}^2 - 1 - \xi \bar{\Phi}^4)^2 + 2\bar{S}^2 \bar{\Phi}^2 (1 - 2\xi \bar{\Phi}^2)^2 \right) & \text{for sh-FHI}, \\ \mu_{\rm S}^4 \left((1 - \bar{\Phi}^4)^2 + 16\bar{S}^2 \bar{\Phi}^6 \right) & \text{for sm-FHI}, \end{cases}$$

$$\text{WHERE:} \begin{cases} \bar{\Phi} = |\Phi|/M \text{ and } \bar{S} = |S|/M & \text{for St- or Sh-FHI}, \\ \bar{\Phi} = |\Phi|/2\sqrt{\mu_{\rm S}M_{\rm S}} & \text{and } \bar{S} = |S|/\sqrt{2\mu_{\rm S}M_{\rm S}} & \text{for Sm-FHI}, \end{cases}$$

and $\xi = M^2/\kappa M_{
m S}$ with $1/7.2 < \xi < 1/4$ °.

2. STRUCTURE

⁹R. Jeannerot et al. (2000)

• W Leads to the SSB of G, Since the SUSY Vacuum is:

$$\langle S \rangle = 0 \ \mbox{and} \ |\langle \bar{\Phi} \rangle| = |\langle \Phi \rangle| = v_{_G} \ \mbox{With} \ v_{_G} = \begin{cases} M & \mbox{for st-Fhi}, \\ \frac{M}{\sqrt{2\xi}} \sqrt{1 - \sqrt{1 - 4\xi}} & \mbox{for sh-Fhi}, \\ \sqrt{\mu_{\rm S}} M_{\rm S} & \mbox{for sm-Fhi}. \end{cases}$$

• W Also Gives Rise to FHI Since There are:

$$\label{eq:F-FLAT} \text{F-FLAT DIRECTION(S)} \ (V_{\rm F}={\rm cst}): \quad \begin{cases} \bar{\Phi}=0 \ \ G \ \text{is Restored} & \text{for st-FHI}, \\ (\text{TOPOLOGICAL DEFECTS MAY BE PRODUCED}) \\ \bar{\Phi}=0 \ \ \text{Or} \ \ \bar{\Phi}=\sqrt{1/2\xi} & \text{for sh-FHI}, \\ \bar{\Phi}=0 \ \ \text{Or} \ \ \bar{\Phi}=1/2\sqrt{6}\bar{S} & \text{for sm-FHI}. \end{cases}$$

3. PICTORIAL REPRESENTATION









• sm-FHI (: Sмоотн FHI)



4. COMPARISONS

IN THE CASES OF ST-FHI AND SH-FHI:

- The $\bar{\Phi}=0\text{-}\mathrm{Direction}$ is a Minimum of V_{SUSY} for |S| Large.
- THE INFLATIONARY VALLEYS ARE CLASSICALLY FLAT.
- THERE IS A CRITICAL POINT ALONG THE INFLATIONARY VALLEYS.

IN THE CASE OF SM-FHI:

- The $\bar{\Phi} = 0$ -Direction is a Maximum of $V_{\rm SUSY}$.
- THE INFLATIONARY VALLEYS ARE NOT CLASSICALLY FLAT.
- THERE IS NO CRITICAL POINT ALONG THE INFLATION-ARY VALLEYS

C. THE INFLATIONARY POTENTIAL

The Inflationary Potential can be Written As: $V_{\rm HI} = V_{\rm HI0} + V_{\rm HIc} + V_{\rm HIS},~$ Where:

• $V_{\rm HI0}$: The Dominant Contribution to $V_{\rm HI}$ Along the F-Flat Direction,

$$V_{\rm HI0} = \begin{cases} \kappa^2 M^4 & \text{for st-FHI}, \\ \kappa^2 M_\xi^4 & \text{for sh-FHI} \quad (M_\xi = M \sqrt{1/4\xi - 1}), \\ \mu_{\rm S}^4 & \text{for sm-FHI}. \end{cases}$$

• $V_{\rm HIc}$: Corrections to $V_{\rm HI0}$ Which Generate the Slope Along the Flat Direction Which Is Necessary For Driving $\sigma = |S|/\sqrt{2}$ Towards The Vacua. $V_{\rm HI0} > 0$ Breaks SUSY and Gives Rise to Logarithmic RCs to $V_{\rm HI}$. In the Case of sm-FHI, the Inflationary Valleys are not Classically Flat and, Thus, There Is No Need of RCs.

$$V_{\rm HIc} = \begin{cases} \frac{\kappa^4 M^4 {\rm N}}{32\pi^2} \left(2\ln \frac{\kappa^2 x M^2}{Q^2} + f_c(x) \right), \ x = \frac{\sigma^2}{M^2} & \text{for st-Fhi}, \\ \frac{\kappa^4 M_{\xi}^4}{16\pi^2} \left(2\ln \frac{\kappa^2 x_{\xi} M_{\xi}^2}{Q^2} + f_c(x_{\xi}) \right), \ x_{\xi} = \frac{\sigma^2}{M_{\xi}^2} & \text{for sh-Fhi}, \\ -2\mu_{\rm s}^6 M_{\rm S}^2 / 27\sigma^4 & \text{for sm-Fhi}, \end{cases}$$

Where $f_c(x) = (x+1)^2 \ln(1+x^{-1}) + (x-1)^2 \ln(1-x^{-1})$

and N the Dimensionality of The Reps To Which $ar{\Phi}$ and Φ belong and Q a RN Scale.

• $V_{\rm HIS}$: SUGRA Corrections to $V_{\rm HI}$ Assuming Minimal Kähler Potential,

$$V_{\rm HIS} = V_{\rm HI0} \frac{\sigma^4}{8m_{\rm P}^4}$$
, where $m_{\rm P} \simeq 2.44 \times 10^{18} \,{\rm GeV}$.

D. THE INFLATIONARY OBSERVABLES

UNDER THE ASSUMPTION THAT THE COSMOLOGICAL SCALES LEAVE THE HORIZON DURING FHI AND ARE NOT REPROCESSED DURING MI, WE CAN EXTRACT:

• The Number of e-foldings $N_{\rm HI*}$ That k_* Suffered During FHI,

$$N_{
m HI*}=~rac{1}{m_{
m P}^2}~\int_{\sigma_{
m f}}^{\sigma_*}~d\sigma~rac{V_{
m HI}}{V_{
m HI}'},~{
m where}~':d/d\sigma~{
m and}$$

- σ_* : The Value of σ When the Scale k_* Crossed Outside the Horizon of FHI,
- $\sigma_{\rm f}$: The Value of σ at the End of FHI, Which can be Found, in the Slow Roll Approximation, from the Condition:

$$\max\{\epsilon(\sigma_{\rm f}), |\eta(\sigma_{\rm f})|\} = 1, \text{ with } \epsilon \simeq \frac{m_{\rm P}^2}{2} \left(\frac{V_{\rm HI}'}{V_{\rm HI}}\right)^2 \text{ and } \eta \simeq m_{\rm P}^2 \frac{V_{\rm HI}''}{V_{\rm HI}}.$$

However, $\sigma_{\rm f} \simeq \begin{cases} \sigma_{\rm c} = M/\sqrt{2} & \text{for st-FHI}, \\ \sigma_{\rm c} = M & \text{for sh-FHI}. \end{cases}$

• The Power Spectrum $P_{\mathcal{R}*}$ of the Curvature Perturbations at the Pivot Scale k_* ,

$$P_{\mathcal{R}*}^{1/2} = \left. \frac{1}{2\sqrt{3}\pi m_{\rm P}^3} \left. \frac{V_{\rm HI}^{3/2}}{|V_{\rm HI}'|} \right|_{\sigma=\sigma_*} \right.$$

• The Spectral Index $n_{\rm s}$ and its Running $\alpha_{\rm s},$

$$n_{\rm s} = 1 + \frac{d\ln P_{\mathcal{R}}}{d\ln k} \bigg|_{\sigma = \sigma_*} = 1 - m_{\rm P}^2 \left. \frac{V_{\rm HI}'}{V_{\rm HI}} (\ln P_{\mathcal{R}})' \right|_{\sigma = \sigma_*} = 1 - 6\epsilon(\sigma_*) + 2\eta(\sigma_*),$$

and
$$\alpha_{\rm s} = \left. \frac{d^2 \ln P_{\mathcal{R}}}{d \ln k^2} \right|_{\sigma = \sigma_*} = 2 \left(4\eta (\sigma_*)^2 - (n_{\rm s} - 1)^2) / 3 - 2\xi(\sigma_*) \text{ with } \xi \simeq m_{\rm P}^4 V_{\rm HI}' V_{\rm HI}'' / V_{\rm HI}^2$$

III. BASICS OF MODULAR INFLATION

A. THE INFLATIONARY POTENTIAL

THE RELEVANT FOR MI PART OF THE POTENTIAL HAS THE FORM¹⁰:

$$V_{
m MI} = V_{
m MI0} - rac{1}{2}m_s^2 s^2 + \cdots,$$
 with:

- s: the Canonically Normalized String Axion With Mass $m_s\sim m_{3/2}\sim 1~{\rm TeV}$ where $m_{3/2}$ is the Gravitino Mass,
- $V_{\rm MI0} = v_s (m_{3/2} m_{\rm P})^2 \Rightarrow V_{\rm MI0}^{1/4} \simeq 3 \times 10^{10} \, {\rm GeV} \, (v_s \sim 1)$. Therefore, the Contribution of MI to $P_{\mathcal{R}}$ is Negligible Since $V_{\rm MI0} \ll V_{\rm HI0}$.



B. DYNAMICS OF THE STRING AXION

The Solution of the EOM of s: $\ddot{s} + 3Hs - m^2s = 0$, is $s = s_i e^{F_s Ht}$ where $F_s = \sqrt{\frac{9}{4} + \left(\frac{m_s}{H}\right)^2 - \frac{3}{2}}$,

- s_i : the Initial Value of s and
- H: THE HUBBLE PARAMETER WHICH IS:
 - During FHI, $H = H_{\rm HI0} = \sqrt{V_{\rm HI0}}/\sqrt{3}m_{\rm P} \sim (10^{10} 10^{12}) \,{\rm GeV} \gg m_s$. Therefore, s Remains Practically Frozen. Note That $V_{\rm MI}$ Remains Unaltered During FHI (and the MD Era After HI) Since s is a Pheudo Nambu Goldstone Boson (It has no K and W)¹¹.
 - During FHI, $H = H_s \simeq \sqrt{V_{\text{MI0}}} / \sqrt{3} m_{\text{P}} = m_s \sqrt{v_s/3} \sim m_s$. For Natural MI We Need: $(V_{\text{MI}} > 0 \)0.5 \le v_s \le 10 \ \Rightarrow \ 2.45 \ge m_s/H_s \ge 0.55 \ \Rightarrow \ 1.37 \ge F_s \ge 0.097$.

¹⁰ P. Binétruy and M.K. Gaillard (1986); F.C. Adams et al. (1993).

¹¹ M. Dine, L. Randall, and S. Thomas (1995); E.J. Chun, K. Dimopoulos and D.H. Lyth (2004).

C. TYPES OF MI

INFLATION CAN BE NOT ONLY OF THE SLOW-ROLL BUT ALSO OF THE FAST-ROLL TYPE SINCE¹²:

$$|\eta_s| = m_{\rm P}^2 \frac{d^2 V_{\rm MI}/ds^2}{V_{\rm MI}} = \frac{m_s^2}{3H_s^2} \simeq \sqrt{\frac{1}{v_s}} \implies \begin{cases} m_s/H_s < \sqrt{3} & \text{or } v_s > 1 \\ m_s/H_s > \sqrt{3} & \text{or } v_s < 1 \end{cases}$$
 Slow-Roll MI, But $\dot{H}_s = \frac{1}{2} \frac{s^2}{s^2} = 10.005 + 0.041 \text{ m} s = 1.15 \text{ m} s = 0.55 \text{ m} s = 1.15 \text{ m} s$

$$\epsilon_s = -\frac{\pi_s}{H_s^2} = F_s^2 \frac{s^2}{2m_{\rm P}^2} \in [0.005, 0.94] \implies \epsilon_s < 1 \text{ for } 0.55 \le m_s/H_s \le 2.45 \text{ and } \langle s \rangle/m_{\rm P} = 1.55 \text{ for } 0.55 \le m_s/H_s \le 2.45 \text{ and } \langle s \rangle/m_{\rm P} = 1.55 \text{ for } 0.55 \le m_s/H_s \le 2.45 \text{ and } \langle s \rangle/m_{\rm P} = 1.55 \text{ for } 0.55 \le m_s/H_s \le 2.45 \text{ and } \langle s \rangle/m_{\rm P} = 1.55 \text{ for } 0.55 \le m_s/H_s \le 2.45 \text{ for } 0.55 \le m_s/H$$

Therefore, We Obtain Accelerated Expansion (Inflation) with $H_s = \text{cst.}$ We Take $s_f = \langle s \rangle = m_P$, Since $\epsilon_s = 1 \iff s_{\rm f} > m_{\rm P}$ for $1.37 \ge F_s \ge 0.097$.

D. NUMBER OF *e*-FOLDINGS

THE TOTAL NUMBER OF *e*-Foldings During MI Can be Found from: $N_{\rm MI} = \frac{1}{F_s} \ln \frac{s_{\rm f}}{s_{\rm i}} \simeq \frac{1}{F_s} \ln \left(\frac{m_{\rm P}}{s_{\rm i}}\right)$.

$$\mathbf{z}^{\text{i}} = \mathbf{x}^{\text{i}} + \mathbf{y}^{\text{i}} +$$

¹²A. Linde (2001)

WE OBSERVE THAT:

- As $s_{
 m i}$ Decreases, The Required m_s/H_s For Obtain-ING $N_{
 m MI} \sim 30$ increases. Also for $s_{
 m i}/m_{
 m P} < 10^{-8}$, WE NEED FAST-ROLL MI.
- For $s_{\rm i}/m_{
 m P}~>~0.1$, It is not Possible to Obtain $N_{\rm MI} \sim 30$

III. OBSERVATIONAL CONSTRAINTS

THE COSMOLOGICAL SCENARIO UNDER CONSIDERATION NEEDS TO SATISFY A NUMBER OF CONSTRAINTS WHICH ARISE FROM:

- (i) The Power Spectrum of the Curvature Perturbations: $P_{\mathcal{R}*}^{1/2} \simeq 4.86 \times 10^{-5}$ at $k_* = 0.002/\mathrm{Mpc}$.
- (ii) The Low Enough Value of $\alpha_{\rm s}$ (In Order to be Consistent with the power-law $\Lambda {\rm CDM}~{\rm Model})^{\rm 13}$: $|\alpha_{\rm s}|\ll 0.01$. We Display Curves for $\alpha_{\rm s}=-0.005~{\rm and}~-0.01$.
- (ii) THE RESOLUTION OF THE HORIZON AND FLATNESS PROBLEMS OF SBB:

$$N_{\rm tot} = N_{\rm HI*} + N_{\rm MI} \simeq \ln \frac{H_0 a_0}{k} + 24.72 + \frac{2}{3} \ln \frac{V_{\rm HI0}^{1/4}}{1 \,\,{\rm GeV}} + \frac{1}{3} \ln \frac{T_{\rm Mrh}}{1 \,\,{\rm GeV}}, \quad {\rm at} \quad k = k_* = 0.002 / {\rm Mpc}.$$

We Assume That $T_{\rm Hrh} < V_{\rm MI0}^{1/4}$ and, Thus, we Obtain Just MD During The Inter-Inflationary Era ($T_{\rm Mrh}$, the Reheat Temperature After MI).

(iii) THE HOMOGENEITY OF THE PRESENT UNIVERSE:

$$s_{
m i}\gg \left.\delta s_{
m i}
ight|_{
m HI}\simeq H_{
m HI0}/2\pi$$
 and $s_{
m i}\gg \left.\delta s_{
m i}
ight|_{
m MI}\simeq H_{
m MI0}/2\pi$

Where $H_{
m HI0}=\sqrt{V_{
m HI0}}/\sqrt{3}m_{
m P}$ and $\delta s_{
m i}$ are the Quantum Fluctuations of s During FHI.

(iv) The Complete Randomization of the String Axion and the Belonging of all the Values of *s* to the Randomization Region with Equal Probability:

$$V_{\rm MI0} \lesssim H_{\rm HI0}^4 \Rightarrow s_i/m_{\rm P} \in (0,1).$$

(v) The Naturalness of MI: $m_s/H_s \lesssim 2.45 ~\Rightarrow~ N_{\rm MI} \gtrsim 0.73 \ln(m_{\rm P}/s_{\rm i}) \sim 3.3$ for $s_{\rm i}/m_{\rm P} \simeq 0.01$.

¹³G. Ballesteros, J.A. Casas, and J.R. Espinosa (2006)



- $au = \ln a/a_0 = -\ln(1+z)$: the Logarithmic Time,
- $\bar{H}=H/H_0:$ the Dimensionless Hubble Parameter,
- $\bar{R}_H = 1/\bar{H}$: the Dimensionless Particle Horizon, $\ln \bar{R}_H \propto 2\tau [1.5\tau]$ for RD [MD] era and $\ln \bar{R}_H =$ cst. for FHI or MI,
- $ar{\lambda}=\lambda/a_0$: the Dimensionless Length Scale, $\ln\lambda\propto au$,
- $\bar{\lambda}_*[\bar{\lambda}_{\mathrm{c}}]$: The Scale Corresponding to k_* [k_{c}].

(vi) The Requirement That the Cosmological Scales (with $k < 0.1/{
m Mpc}^{14}$):

- Leave the Horizon During FHI $N_{
 m HI*}\gtrsim N_{
 m tot}(k=0.002/{
 m Mpc})-N_{
 m tot}(k=0.1/{
 m Mpc})=3.9$ and
- Do Not Re-Enter the Horizon Before The Onset Of $\rm MI^{15}$ $N_{\rm HI*}\gtrsim N_{\rm HIc},$ Where:

$$1 = \frac{k_{\rm c}}{H_s a_s} = \frac{H_{\rm c} a_{\rm c}}{H_s a_{\rm HIf}} \frac{a_{\rm HIf}}{a_s} = e^{-N_{\rm HIc}} \left(\frac{V_{\rm HI0}}{V_{\rm MI0}}\right)^{1/2-1/3} \Rightarrow N_{\rm HIc} = \frac{1}{6} \ln \frac{V_{\rm HI0}}{V_{\rm MI0}}$$

Therefore, all in all we have: $N_{\rm HI*} \gtrsim N_{\rm HI*}^{\rm min} \simeq 3.9 + \frac{1}{6} \ln \frac{V_{\rm HI0}}{V_{\rm MI0}} \sim 10.$

¹⁴U. Seljak, A. Slosar, and P. McDonald (2006).

¹⁵C.P. Burgess et al. (2005).

IV. NUMERICAL RESULTS

We take $T_{\rm Mrh} = 1 \text{ GeV}$ and $m_{3/2} = m_s = 1 \text{ TeV}$ through out. Also, We fix $s_i/m_{\rm P} = 0.01$ Since (i) $s_i < \langle s \rangle \simeq m_{\rm P}$ and so, the Extra Terms of V_{MI0} are Irrelevant (ii) Ensures a Large Available Parameter Space For n = 0.958 and $v_g \simeq M_{\rm GUT}$ (This Choice Though Signalises a Very Mild Tunning). **A.** Standard-FHI

1. Set-up

We consider $G = SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ (N = 2).

If $\overline{\Phi}$ and Φ are ${
m SU}(2)_{
m R}$ Doublets With $B-L=-1,\ 1$ Respectively, no Cosmic Strings Are Produced.

2. ALLOWED REGIONS BY ALL THE CONSTRAINTS



For $n_{\rm s} = 0.958$, We Obtain $0.004 \lesssim \kappa \lesssim 0.14$, $0.79 \lesssim v_g/10^{16} \text{ GeV} \lesssim 1.08$, $-0.002 \gtrsim \alpha_{\rm s} \gtrsim -0.01$ and $0.64 \lesssim m_s/H_s \lesssim 0.77$.



For $n_{
m s}=0.958$, We Obtain $10.\lesssim N_{
m HI*}\lesssim 21.7$ and $35\gtrsim N_{
m MI}\gtrsim 24.$

- B. SHIFTED FHI
- 1. <u>Set-up</u>

We Adopt the Pati-Salam Gauge Group $G = SU(4)_c \times SU(2)_L \times SU(2)_R$ and Fix $M_S = 5 \times 10^{17} \text{ GeV}$.

2. ALLOWED REGIONS

The Results are Quite Similar to These for st-FHI (The Bounds on ξ do not cut out any Slice of the Available Parameter Space). But We Can Obtain $v_g = M_{\rm GUT}$.



For $n_{\rm s} = 0.958$ and $v_{_G} = M_{\rm GUT}$, We Obtain $\kappa \simeq 0.01, \ N_{\rm HI*} \simeq 21., \ |\alpha_{\rm s}| = 0.0018, \ m_s/H_s \simeq 0.77$ and $N_{\rm MI} \simeq 24.3$. B. Smooth FHI

1. <u>Set-up</u>

WE DO NOT INCLUDE RADIATIVE CORRECTIONS INTO OUR COMPUTATION, AND SO THERE IS NO SPECIFIC GUT.

- 2. ALLOWED REGIONS
 - In Contrast to st-FHI and sh-FHI, $|lpha_{
 m s}|$ is Considerably Enhanced in the Case of SM-FHI for $v_{_G}\sim M_{
 m GUT}$
 - In the Case of SM-FHI, SUGRA Corrections Play an Important Role for Every $M_{\rm S}$ in the Allowed Region Whereas They Become More and More Significant as κ Increases Above 0.01 in the Cases of st-FHI and sh-FHI.
 - Contrary to st-FHI and sh-FHI, the Constraint $N_{\rm HI*}\gtrsim N_{\rm HI*}^{\rm min}$ does not Restrict the Parameters, Since it is Overshadowed by the Constraint of $n_{\rm s}\simeq 0.926$.



For $n_{\rm s} = 0.958$ and $v_{\rm g} = M_{\rm GUT}$, $M_{\rm S}/(5 \times 10^{17} \,{\rm GeV}) \simeq 0.87, \ N_{\rm HI*} \simeq 18, |\alpha_{\rm s}| = 0.005, m_s/H_s \simeq 0.72$ and $N_{\rm MI} \simeq 27.8.$

V. CONCLUSIONS

We Presented two-Stage Inflationary Models in Which a Superheavy Scale FHI is Followed by an Intermediate Scale MI. We Confronted These Models With the Data on $P_{\mathcal{R}}$ and $n_{\rm s}$ Within the power-law Λ CDM Model. We Showed That These Restrictions Can be met, Provided That $N_{\rm HI*}$ is Restricted to Rather Low Values (~ 20). For Central Values of $P_{\mathcal{R}}$ and $n_{\rm s}$, We Found That:

$$v_{G} \begin{cases} < M_{\rm GUT} \text{ and } 10. \lesssim N_{\rm HI*} \lesssim 21.7 & \text{for st-FHI}, \\ = M_{\rm GUT} \text{ and } N_{\rm HI*} \simeq 21 & \text{for sh-FHI}, \\ = M_{\rm GUT} \text{ and } N_{\rm HI*} \simeq 18 & \text{for sm-FHI}. \end{cases}$$

In all cases, MI of the Slow-Roll Type With $m_s/H_s \sim 0.6 - 0.8$ Naturally Produces $N_{\rm MI} = N_{\rm tot} - N_{\rm HI*} \simeq (20 - 30)$. Therefore, MI Complements Successfully FHI.

VI. FUTURE DIRECTIONS

- What Happens IF s Obtains a Mass Term During FHI?
- WE CAN INCREASE THE REHEAT TEMPERATURE AFTER COMPLEMENTARY INFLATION?
- WE CAN REALIZE A MECHANISM OF BARYOGENESIS WITHIN THIS FRAMEWORK?
- We can Produce the Required $N_{
 m MI} = N_{
 m tot} N_{
 m HI*} \simeq (20-30)$ Through a Thermal Inflation?
- ARE THERE OBSERVATIONAL CONSEQUENCES?