F-Term Hybrid Inflation
Followed by Modular Inflation

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Based on:
G. Lazarides and C.P.
'Reducing the Spectral Index in F-Term Hybrid Inflation
Through a Complementary Modular Inflation'
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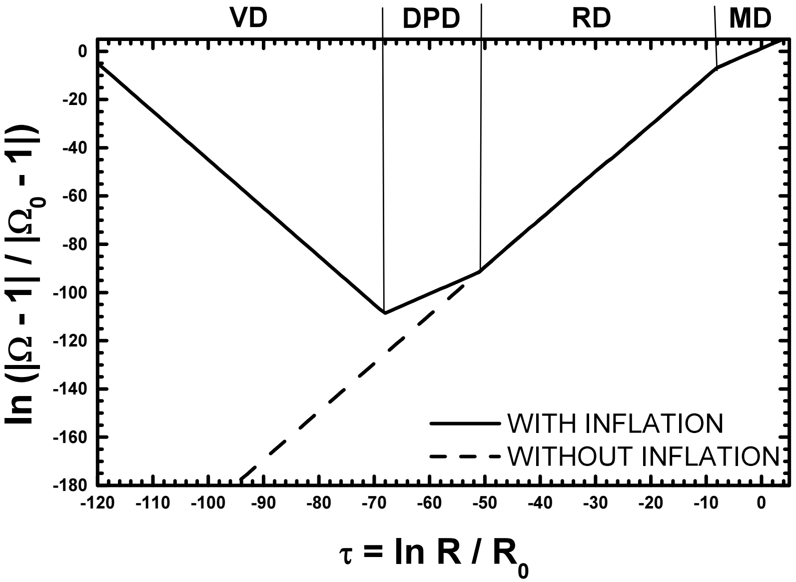
Outline:

- Motivation
- Models of F-Term Hybrid Inflation (FHI)
- The Basics of Modular Inflation (MI)
- Observational Constraints
- Numerical Results
- Conclusions

0. INTRODUCTION TO INFLATION

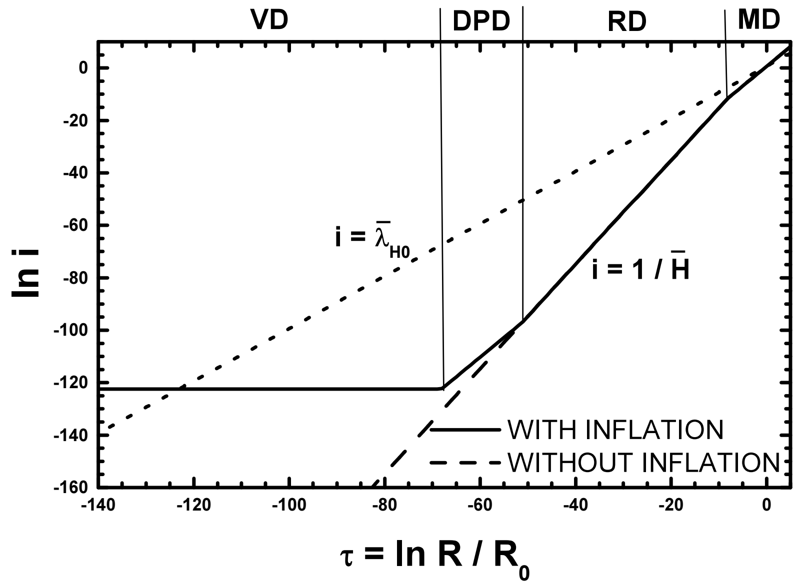
A. PROBLEMS OF STANDARD BIG BANG COSMOLOGY

● THE FLATNESS PROBLEM



WE HAVE $\ln|\Omega-1| \propto 6\boldsymbol{\tau}[4\boldsymbol{\tau}]$ FOR RD [MD] BUT $\ln|\Omega-1| \propto -2\boldsymbol{\tau}$ FOR VD. NOTE THAT $H \propto R^{-2}[R^{-3/2}]$ FOR RD [MD] BUT $H =$CST FOR VD.

● THE HORIZON PROBLEM



A LENGHT $\lambda = 2\pi R/k$ IS INSIDE [OUTSIDE] THE HORIZON $1/$ WHEN $\lambda < 1/H \Leftrightarrow k > RH$ [$\lambda > 1/H \Leftrightarrow k < RH$]. IN ORDER TO RESOLVE THE HORIZON PROBLEM WE NEED $d(\lambda H)/dt > 0 \Leftrightarrow \ddot{R} > 0$ (NOTE THAT $\lambda \propto R$).

B. INFLATION AND THE INFLATON

FROM THE EQUATIONS OF THE COSMOLOGICAL EVOLUTION, WE DEDUCE THAT ACCELERATION OF TH UNIVERSE MEANS:

$$\frac{\ddot{R}}{R} = (1-\epsilon)H^2 = -\frac{1}{6m_{\rm P}}(1+3w)\rho > 0 \quad \Leftrightarrow \quad \begin{cases} \epsilon < 1 & \text{WHERE} \quad \epsilon = -\dot{H}/H^2, \\ w < -1/3 & \text{WHERE} \quad w = P/\rho. \end{cases}$$

Since the Density ρ_ϕ and the Pressure P_ϕ of a omogenous Scalar Field $\phi(t)$ are:

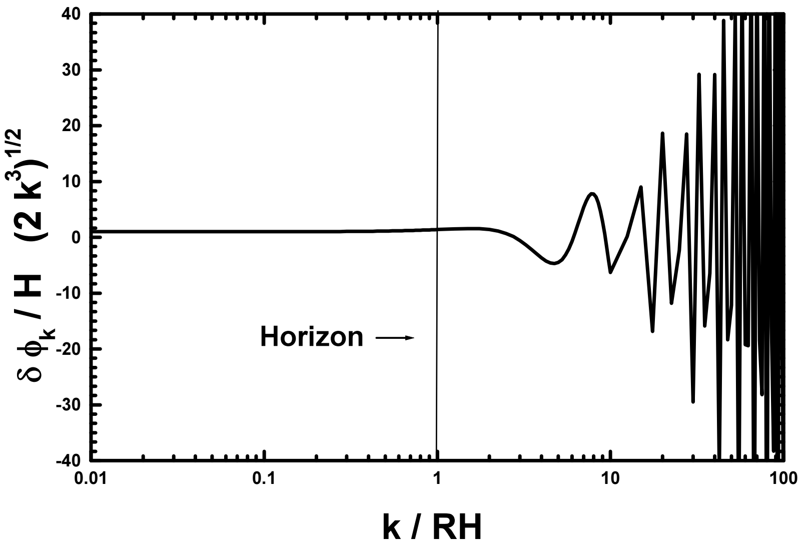
$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \ \text{ and } \ P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \ \text{ We Can Obtain } \ P_\phi = -\rho_\phi \ (w=-1) \ \text{ if } \ \dot{\phi} \ll V(\phi).$$

or $H^2 = \rho_\phi/3m_{\rm P}^2 = {\rm cst.}$ As a Consequence $R(t) = R_{\rm i}e^{\Delta N_e}$ while $T(t) = T_{\rm i}e^{-\Delta N_e}$ Where $\Delta N_e = H(t-t_{\rm i})$ is the Number of e-Foldings During Inflation for $t > t_{\rm i}$.

If in Addition we Have $\dot{\phi} \gg \ddot{\phi} \Leftrightarrow \eta < 1$, Where $\eta = m_{\rm P}^2 d^2V(\phi)/d\phi^2/V$ We Obtain "Slow Roll" Inflation and the Equation oF Motion (: EOM) of ϕ:

$$\ddot{\phi} + 3H\dot{\phi} + dV(\phi)/d\phi = 0 \ \text{ Can be Written as } \ -3H\dot{\phi} = dV(\phi)/d\phi.$$

C. Primordial Curvature Perturbations



Expanding in Fourier Series the Perturbations $\delta\phi(\mathbf{x},t)$ of ϕ, $\delta\phi(\mathbf{x},t) = \phi(\mathbf{x},t) - \phi(t)$,

$$\delta\phi(\mathbf{x},t) = \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} e^{i\mathbf{kx}}\,\delta\phi_{\mathbf{k}}(t),$$

We Obtain a Power Spectrum of the mode $\delta\phi_{\mathbf{k}}$, $P_\phi^{1/2} = (k^3/2\pi)^{1/2}\delta\phi_{\mathbf{k}}(k=RH)$ Which Results to a Power Spectrum of Curvature Perturbations $P_{\mathcal{R}}^{1/2} = H^2/\dot{\phi}P_\phi^{1/2}$. This is Related to the Quadripole Anisotropy $\Delta T/T$ of CMB Measured by WMAP.

I. MOTIVATION

A. WMAP3 AND FHI

FITTING THE WMAP3 DATA WITH THE STANDARD POWER-LAW COSMOLOGICAL MODEL ΛCDM, ONE OBTAINS[1] THAT, AT THE PIVOT SCALE $k_* = 0.002/\mathrm{Mpc}$,

$$n_{\rm s} = 0.958 \pm 0.016 \ \Rightarrow \ 0.926 \lesssim n_{\rm s} \lesssim 0.99 \ \text{(95\% c.l.)}$$

THESE RESULTS BRING UNDER CONSIDERABLE STRESS A CLASS OF SUSY MODELS OF FHI, REALIZED AT (OR VERY CLOSE TO) $M_{\rm GUT} = 2.86 \times 10^{16}$ GeV WHICH PREDICTS (FOR $N_{\rm tot} = N_{\rm HI*} = 50$)[2]:

$$n_{\rm s} \sim 0.98 \ \text{or even} \ n_{\rm s} \sim 1,$$

IF SUGRA CORRECTIONS (WITH CANONINAL KÄHLER POTENTIAL) ARE INCLUDED[3].

B. PROPOSED SOLUTIONS

- UTILIZATION OF A QUASI-CANONINAL KÄHLER POTENTIAL WHICH CAN GENERATE A MAXIMUM ON THE INFLATIONARY PATH (HILLTOP INFLATION). INDISPENSABLE TUNING OF THE INITIAL CONDITIONS (~ 0.01) IS NEEDED[4].
- INCLUSION OF A SMALL CONTRIBUTION TO $P_{\mathcal{R}}$ FROM COSMIC STRINGS, WHICH, HOWEVER, REQUIRES $M \ll M_{\rm GUT}$[5].

C. COMPLEMENTARY INFLATION

OUR PROPOSAL IS BASED ON THE OBSERVATION THAT $n_{\rm S}$ WITHIN FHI GENERALLY DECREASES WITH THE NUMBER OF e-FOLDINGS, $N_{\rm HI*}$, THAT k_* SUFFERED DURING FHI: E.G., FOR ST-FHI WITH SUPERPOTENTIAL $W = \kappa S\left(\bar\Phi\Phi - M^2\right)$ (SEE BELOW):

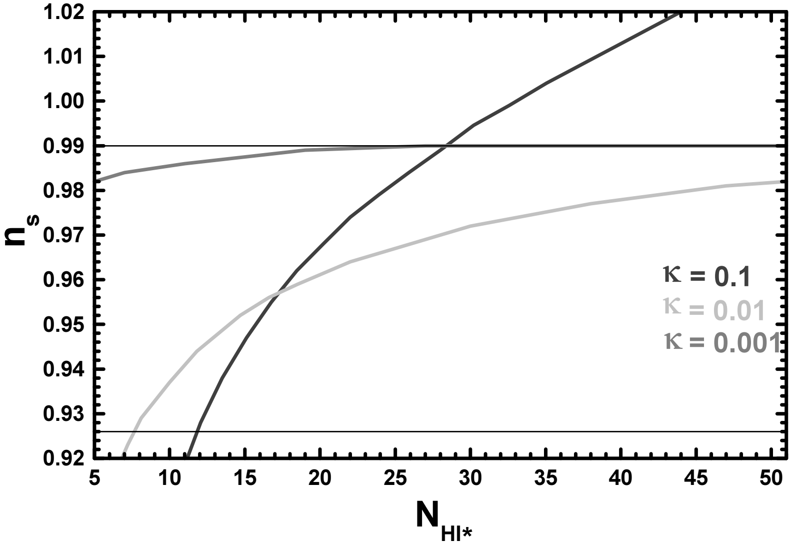
[1] *D.N. Spergel et al. (2006)*

[2] *G.R. Dvali, Q. Shafi, and R.K. Schaefer (1994); G. Lazarides, R.K. Schaefer, and Q. Shafi (1997).*

[3] *V.N. Şenoğuz and Q. Shafi (2003).*

[4] *L. Boubekeur and D. Lyth (2005); M. Bastero-Gil, S.F. King, and Q. Shafi (2006); B. Garbrecht, C. P., and A. Pilaftsis (2006); M.U. Rehman, V.N. Şenoğuz, and Q. Shafi (2006).*

[5] *R.A. Battye, B. Garbrecht, and A. Moss (2006); R. Jeannerot and M. Postma (2005); J. Rocher and M. Sakellariadou (2005).*



WE OBSERVE THAT FOR RELATIVELY LARGE κ $(\simeq 0.01, 0.1)$ AND $N_{\rm HI*} \sim (15-20)$ WE CAN OBTAIN $n_{\rm s} \simeq 0.96$. THE RESIDUAL NUMBER OF e-FOLDINGS $N_{\rm tot} - N_{\rm HI*}$ (REQUIRED FOR THE RESOLUTION OF THE HORIZON AND FLATNESS PROBLEMS OF SBB) CAN BE OBTAINED BY A SECOND STAGE OF INFLATION REALIZED AT A LOWER SCALE.WE CALL THIS TYPE OF INFLATION COMPLEMENTARY INFLATION. WE CAN SHOW THAT MI CAN NATURALLY PLAY THIS ROLE.

II. MODELS OF FHI

A. THE RELEVANT SUPERPOTENTIAL

THE FHI CAN BE REALIZED ADOPTING ONE OF THE SUPERPOTENTIALS[6 7 8]:

$$W = \begin{cases} \kappa S\left(\bar\Phi\Phi - M^2\right) & \text{for standard FHI (: st-FHI)},\\ \kappa S\left(\bar\Phi\Phi - M^2\right) - S\frac{(\bar\Phi\Phi)^2}{M_{\rm S}^2} & \text{for shifted FHI (: sh-FHI)},\\ S\left(\frac{(\bar\Phi\Phi)^2}{M_{\rm S}^2} - \mu_{\rm S}^2\right) & \text{for smooth FHI (: sm-FHI)},\end{cases}$$ WHERE:

[6] sm-FHI: *G.R. Dvali, Q. Shafi, and R.K. Schaefer (1994)*

[7] sh-FHI: *R. Jeannerot, S. Khalil, G. Lazarides and Q. Shafi (2000)*

[8] sm-FHI: *G. Lazarides and C. Panagiotakopoulos (1995); R. Jeannerot, S. Khalil, and G. Lazarides (2001)*

- $\bar{\Phi}$ AND Φ: PAIR OF LEFT HANDED SUPERFIELDS BELONGING TO NON-TRIVIAL CONJUGATE REPS OF A GAUGE GROUP G AND REDUCING ITS RANK BY THEIR VEVS (WATERFALL FIELDS),
- $M_{\rm S} \sim 5 \times 10^{17}~{\rm GeV}$: THE STRING SCALE,
- κ AND $M,~\mu_{\rm S}~(\sim M_{\rm GUT})$: POSITIVE PARAMETERS.

$$W \ni \begin{cases} \text{RENORMALIZABLE TERMS CONSISTENT WITH} & \\ U(1)_R : S \;\to\; e^{i\alpha}\, S,\; \bar{\Phi}\Phi \;\to\; \bar{\Phi}\Phi,\; W \to\; e^{i\alpha}\, W & \text{FOR ST-FHI}, \\ \text{LEADING NON-RENORMALIZABLE TERM} & \text{FOR SH-FHI}, \\ Z_2 \;\text{ INVARIANT TERMS UNDER WHICH }\; \Phi \to -\Phi & \text{FOR SM-FHI}. \end{cases}$$

B. THE SUSY POTENTIAL

1. DERIVATION

THE SUSY POTENTIAL INCLUDES 2 CONTRIBUTIONS: $V_{\rm SUSY} = V_{\rm F} + V_{\rm D}$, WHERE

- D-TERM CONTRIBUTION: $V_{\rm D} = 0$, WITH $|\bar{\Phi}| = |\Phi|$.
- F-TERM CONTRIBUTION: $V_{\rm F} = \begin{cases} \kappa^2 M^4 \left((\bar{\Phi}^2 - 1)^2 + 2\bar{S}^2\bar{\Phi}^2\right) & \text{FOR ST-FHI}, \\ \kappa^2 M^4 \left((\bar{\Phi}^2 - 1 - \xi\bar{\Phi}^4)^2 + 2\bar{S}^2\bar{\Phi}^2(1 - 2\xi\bar{\Phi}^2)^2\right) & \text{FOR SH-FHI}, \\ \mu_{\rm S}^4 \left((1 - \bar{\Phi}^4)^2 + 16\bar{S}^2\bar{\Phi}^6\right) & \text{FOR SM-FHI}, \end{cases}$

$$\text{WHERE: } \begin{cases} \bar{\Phi} = |\Phi|/M \;\text{ AND }\; \bar{S} = |S|/M & \text{FOR ST- OR SH-FHI}, \\ \bar{\Phi} = |\Phi|/2\sqrt{\mu_{\rm S} M_{\rm S}} \;\text{ AND }\; \bar{S} = |S|/\sqrt{2\mu_{\rm S} M_{\rm S}} & \text{FOR SM-FHI}, \end{cases}$$

AND $\xi = M^2/\kappa M_{\rm S}$ WITH $1/7.2 < \xi < 1/4$ [9].

2. STRUCTURE

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[9] *R. Jeannerot et al. (2000)*

- W LEADS TO THE SSB OF G, SINCE THE SUSY VACUUM IS:

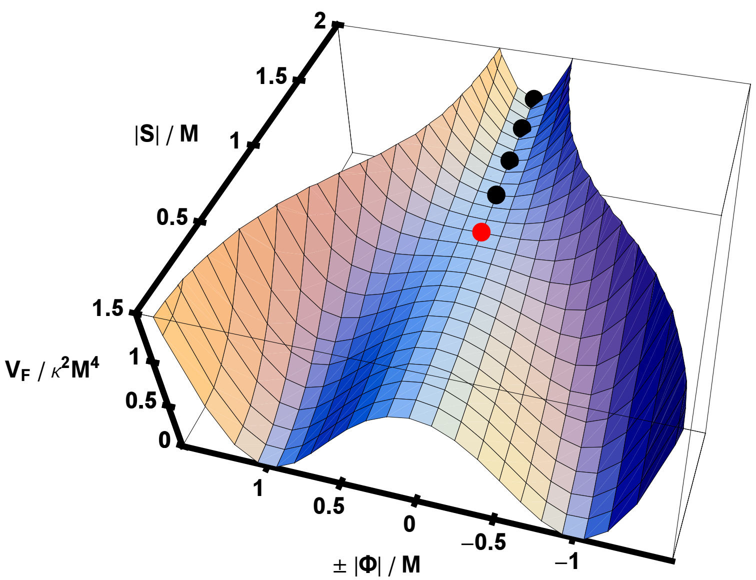
$$\langle S\rangle = 0 \text{ AND } |\langle\bar\Phi\rangle| = |\langle\Phi\rangle| = v_G \text{ WITH } v_G = \begin{cases} M & \text{FOR ST-FHI,} \\ \frac{M}{\sqrt{2\xi}}\sqrt{1-\sqrt{1-4\xi}} & \text{FOR SH-FHI,} \\ \sqrt{\mu_{\rm S} M_{\rm S}} & \text{FOR SM-FHI.} \end{cases}$$

- W ALSO GIVES RISE TO FHI SINCE THERE ARE:

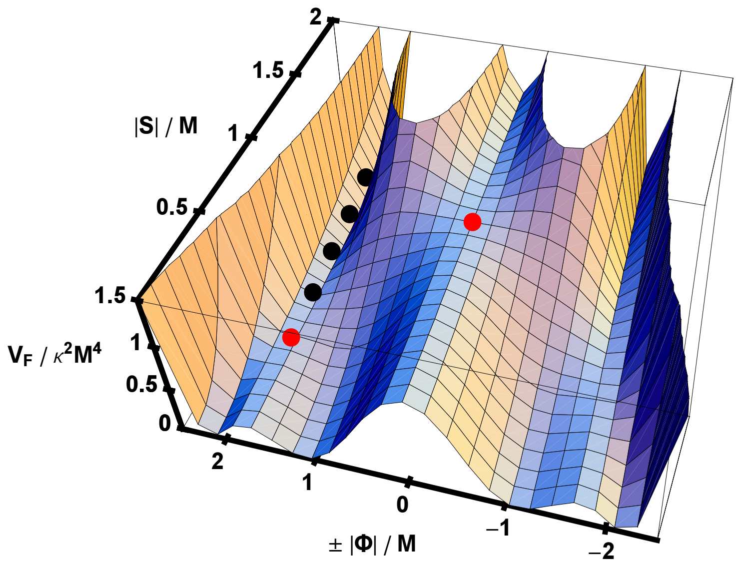
$$\text{F-FLAT DIRECTION(S) } (V_{\rm F} = {\rm cst}): \begin{cases} \bar\Phi = 0 \;\; G \text{ IS RESTORED} & \text{FOR ST-FHI,} \\ \text{(TOPOLOGICAL DEFECTS MAY BE PRODUCED)} & \\ \bar\Phi = 0 \text{ OR } \bar\Phi = \sqrt{1/2\xi} & \text{FOR SH-FHI,} \\ \bar\Phi = 0 \text{ OR } \bar\Phi = 1/2\sqrt{6}\bar S & \text{FOR SM-FHI.} \end{cases}$$

3. PICTORIAL REPRESENTATION

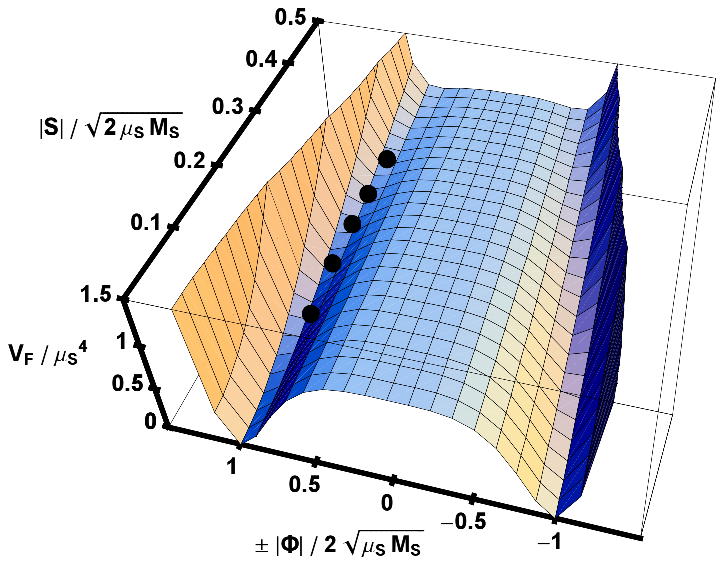
- ST-FHI (: STANDARD FHI)



- SH-FHI (: SHIFTED FHI)



● sm-FHI (: Smooth FHI)



4. Comparisons

In the Cases of st-FHI and sh-FHI:

- The $\bar{\Phi} = 0$-Direction is a Minimum of $V_{\rm SUSY}$ for $|S|$ Large.
- The Inflationary Valleys are Classically Flat.
- There is a Critical Point Along the Inflationary Valleys.

In the Case of sm-FHI:

- The $\bar{\Phi} = 0$-Direction is a Maximum of $V_{\rm SUSY}$.
- The Inflationary Valleys are not Classically Flat.
- There is no Critical Point Along the Inflationary Valleys

C. THE INFLATIONARY POTENTIAL

THE INFLATIONARY POTENTIAL CAN BE WRITTEN AS: $V_{\rm HI} = V_{\rm HI0} + V_{\rm HIc} + V_{\rm HIS},$ WHERE:

- $V_{\rm HI0}$: THE DOMINANT CONTRIBUTION TO $V_{\rm HI}$ ALONG THE F-FLAT DIRECTION,

$$V_{\rm HI0} = \begin{cases} \kappa^2 M^4 & \text{for st-FHI}, \\ \kappa^2 M_\xi^4 & \text{for sh-FHI} \quad (M_\xi = M\sqrt{1/4\xi - 1}), \\ \mu_{\rm S}^4 & \text{for sm-FHI}. \end{cases}$$

- $V_{\rm HIc}$: CORRECTIONS TO $V_{\rm HI0}$ WHICH GENERATE THE SLOPE ALONG THE FLAT DIRECTION WHICH IS NECESSARY FOR DRIVING $\sigma = |S|/\sqrt{2}$ TOWARDS THE VACUA. $V_{\rm HI0} > 0$ BREAKS SUSY AND GIVES RISE TO LOGARITHMIC RCS TO $V_{\rm HI}$. IN THE CASE OF SM-FHI, THE INFLATIONARY VALLEYS ARE NOT CLASSICALLY FLAT AND, THUS, THERE IS NO NEED OF RCS.

$$V_{\rm HIc} = \begin{cases} \frac{\kappa^4 M^4 {\sf N}}{32\pi^2}\left(2\ln\frac{\kappa^2 x M^2}{Q^2} + f_c(x)\right),\ x = \frac{\sigma^2}{M^2} & \text{for st-FHI}, \\ \frac{\kappa^4 M_\xi^4}{16\pi^2}\left(2\ln\frac{\kappa^2 x_\xi M_\xi^2}{Q^2} + f_c(x_\xi)\right),\ x_\xi = \frac{\sigma^2}{M_\xi^2} & \text{for sh-FHI}, \\ -2\mu_{\rm s}^6 M_{\rm S}^2/27\sigma^4 & \text{for sm-FHI}, \end{cases}$$

$$\text{WHERE } f_c(x) = (x+1)^2\ln(1+x^{-1}) + (x-1)^2\ln(1-x^{-1})$$

AND N THE DIMENSIONALITY OF THE REPS TO WHICH $\bar\Phi$ AND Φ BELONG AND Q A RN SCALE.

- $V_{\rm HIS}$: SUGRA CORRECTIONS TO $V_{\rm HI}$ ASSUMING MINIMAL KÄHLER POTENTIAL,

$$V_{\rm HIS} = V_{\rm HI0}\frac{\sigma^4}{8m_{\rm P}^4}, \quad \text{where } m_{\rm P} \simeq 2.44\times 10^{18}\ {\rm GeV}.$$

D. THE INFLATIONARY OBSERVABLES

UNDER THE ASSUMPTION THAT THE COSMOLOGICAL SCALES LEAVE THE HORIZON DURING FHI AND ARE NOT REPROCESSED DURING MI, WE CAN EXTRACT:

- The Number of e-foldings $N_{\rm HI*}$ That k_* Suffered During FHI,

$$N_{\rm HI*}=\;\frac{1}{m_{\rm P}^2}\;\int_{\sigma_{\rm f}}^{\sigma_*}\,d\sigma\,\frac{V_{\rm HI}}{V'_{\rm HI}},\ \text{where}\ {}':d/d\sigma\ \ \text{and}$$

  - σ_*: the Value of σ When the Scale k_* Crossed Outside the Horizon of FHI,
  - $\sigma_{\rm f}$: the Value of σ at the End of FHI, Which can be Found, in the Slow Roll Approximation, from the Condition:

$$\max\{\epsilon(\sigma_{\rm f}),|\eta(\sigma_{\rm f})|\}=1,\ \ \text{with}\ \ \epsilon\simeq\frac{m^2_{\rm P}}{2}\left(\frac{V'_{\rm HI}}{V_{\rm HI}}\right)^2\ \ \text{and}\ \ \eta\simeq m^2_{\rm P}\,\frac{V''_{\rm HI}}{V_{\rm HI}}\cdot$$

However, $\sigma_{\rm f}\simeq\begin{cases}\sigma_{\rm c}=M/\sqrt{2} & \text{for st-FHI},\\ \sigma_{\rm c}=M & \text{for sh-FHI}.\end{cases}$

- The Power Spectrum $P_{\mathcal{R}*}$ of the Curvature Perturbations at the Pivot Scale k_*,

$$P^{1/2}_{\mathcal{R}*}=\;\frac{1}{2\sqrt{3}\,\pi m_{\rm P}^3}\;\;\frac{V_{\rm HI}^{3/2}}{|V'_{\rm HI}|}\right|_{\sigma=\sigma_*}\cdot$$

- The Spectral Index $n_{\rm s}$ and its Running $\alpha_{\rm s}$,

$$n_{\rm s}=1+\left.\frac{d\ln P_{\mathcal{R}}}{d\ln k}\right|_{\sigma=\sigma_*}=1-m^2_{\rm P}\,\frac{V'_{\rm HI}}{V_{\rm HI}}(\ln P_{\mathcal{R}})'\Big|_{\sigma=\sigma_*}=1-6\epsilon(\sigma_*)\;+\;2\eta(\sigma_*),$$

$$\text{and}\ \ \alpha_{\rm s}=\left.\frac{d^2\ln P_{\mathcal{R}}}{d\ln k^2}\right|_{\sigma=\sigma_*}=2\left(4\eta(\sigma_*)^2-(n_{\rm s}-1)^2\right)/3-2\xi(\sigma_*)\ \ \text{with}\ \ \xi\simeq m^4_{\rm P}\,V'_{\rm HI}V'''_{\rm HI}/V^2_{\rm HI}$$

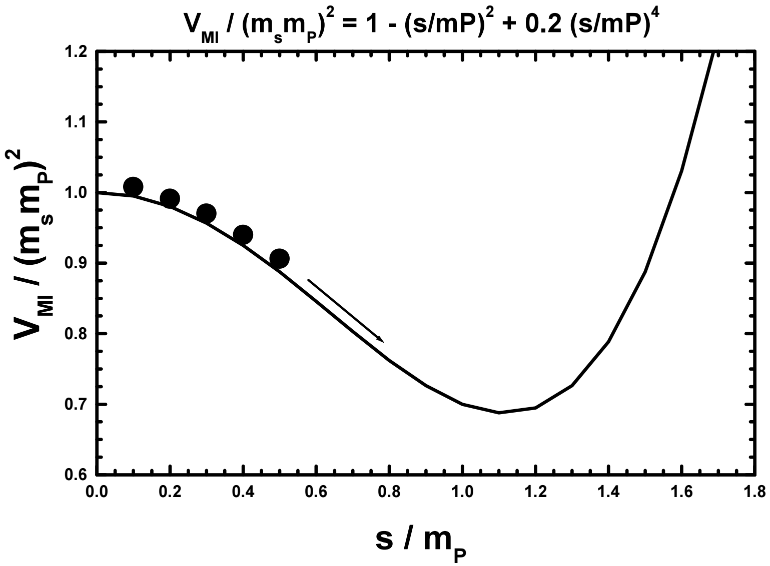
III. BASICS OF MODULAR INFLATION

A. THE INFLATIONARY POTENTIAL

THE RELEVANT FOR MI PART OF THE POTENTIAL HAS THE FORM[10]:

$$V_{\rm MI} = V_{\rm MI0} - \frac{1}{2} m_s^2 s^2 + \cdots, \text{ WITH:}$$

- s: THE CANONICALLY NORMALIZED STRING AXION WITH MASS $m_s \sim m_{3/2} \sim 1~{\rm TeV}$ WHERE $m_{3/2}$ IS THE GRAVITINO MASS,
- $V_{\rm MI0} = v_s (m_{3/2} m_{\rm P})^2 \;\Rightarrow\; V_{\rm MI0}^{1/4} \simeq 3\times 10^{10}~{\rm GeV}~(v_s \sim 1)$. THEREFORE, THE CONTRIBUTION OF MI TO $P_{\mathcal R}$ IS NEGLIGIBLE SINCE $V_{\rm MI0} \ll V_{\rm HI0}$.



B. DYNAMICS OF THE STRING AXION

THE SOLUTION OF THE EOM OF s: $\ddot s + 3H s - m^2 s = 0$, IS $s = s_{\rm i} e^{F_s H t}$ WHERE $F_s = \sqrt{\frac{9}{4} + \left(\frac{m_s}{H}\right)^2} - \frac{3}{2}$,

- $s_{\rm i}$: THE INITIAL VALUE OF s AND
- H: THE HUBBLE PARAMETER WHICH IS:
  - DURING FHI, $H = H_{\rm HI0} = \sqrt{V_{\rm HI0}}/\sqrt{3} m_{\rm P} \sim (10^{10} - 10^{12})~{\rm GeV} \gg m_s$. THEREFORE, s REMAINS PRACTICALLY FROZEN. NOTE THAT $V_{\rm MI}$ REMAINS UNALTERED DURING FHI (AND THE MD ERA AFTER HI) SINCE s IS A PHEUDO NAMBU GOLDSTONE BOSON (IT HAS NO K AND W)[11].
  - DURING FHI, $H = H_s \simeq \sqrt{V_{\rm MI0}}/\sqrt{3} m_{\rm P} = m_s \sqrt{v_s/3} \sim m_s$.
    FOR NATURAL MI WE NEED: $(V_{\rm MI} > 0~)0.5 \le v_s \le 10 \;\;\Rightarrow\;\; 2.45 \ge m_s/H_s \ge 0.55 \;\;\Rightarrow\;\; 1.37 \ge F_s \ge 0.097.$

[10] *P. Binétruy and M.K. Gaillard (1986); F.C. Adams et al. (1993).*
[11] *M. Dine, L. Randall, and S. Thomas (1995); E.J. Chun, K. Dimopoulos and D.H. Lyth (2004).*

C. TYPES OF MI

INFLATION CAN BE NOT ONLY OF THE SLOW-ROLL BUT ALSO OF THE FAST-ROLL TYPE SINCE[12]:

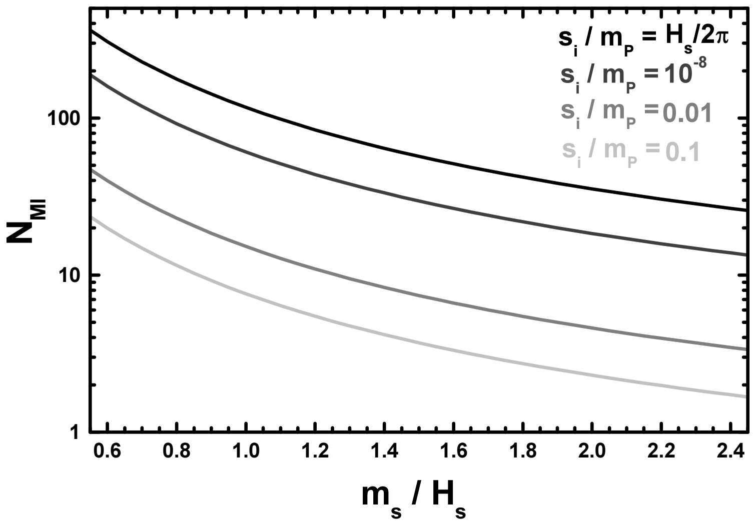
$$|\eta_s| = m_{\rm P}^2 \frac{d^2V_{\rm MI}/ds^2}{V_{\rm MI}} = \frac{m_s^2}{3H_s^2} \simeq \sqrt{\frac{1}{v_s}} \;\Rightarrow\; \begin{cases} m_s/H_s < \sqrt{3} \;\text{ or }\; v_s > 1 & \text{Slow-Roll MI,} \\ m_s/H_s > \sqrt{3} \;\text{ or }\; v_s < 1 & \text{Fast-Roll MI,} \end{cases} \quad \text{But}$$

$$\epsilon_s = -\frac{\dot H_s}{H_s^2} = F_s^2 \frac{s^2}{2m_{\rm P}^2} \in [0.005, 0.94] \;\Rightarrow\; \epsilon_s < 1 \;\text{ For }\; 0.55 \leq m_s/H_s \leq 2.45 \;\text{ And }\; \langle s\rangle/m_{\rm P} = 1.$$

THEREFORE, WE OBTAIN ACCELERATED EXPANSION (:INFLATION) WITH $H_s = {\rm cst}$. WE TAKE $s_{\rm f} = \langle s\rangle = m_{\rm P}$, SINCE $\epsilon_s = 1 \;\Leftrightarrow\; s_{\rm f} > m_{\rm P}$ FOR $1.37 \geq F_s \geq 0.097$.

D. NUMBER OF e-FOLDINGS

THE TOTAL NUMBER OF e-FOLDINGS DURING MI CAN BE FOUND FROM: $N_{\rm MI} = \frac{1}{F_s}\ln\frac{s_{\rm f}}{s_{\rm i}} \simeq \frac{1}{F_s}\ln\left(\frac{m_{\rm P}}{s_{\rm i}}\right).$



WE OBSERVE THAT:

- AS $s_{\rm i}$ DECREASES, THE REQUIRED m_s/H_s FOR OBTAINING $N_{\rm MI} \sim 30$ INCREASES. ALSO FOR $s_{\rm i}/m_{\rm P} < 10^{-8}$, WE NEED FAST-ROLL MI.
- FOR $s_{\rm i}/m_{\rm P} > 0.1$, IT IS NOT POSSIBLE TO OBTAIN $N_{\rm MI} \sim 30$

[12] *A. Linde (2001)*

III. OBSERVATIONAL CONSTRAINTS

THE COSMOLOGICAL SCENARIO UNDER CONSIDERATION NEEDS TO SATISFY A NUMBER OF CONSTRAINTS WHICH ARISE FROM:

(i) THE POWER SPECTRUM OF THE CURVATURE PERTURBATIONS: $P^{1/2}_{\mathcal{R}*} \simeq 4.86 \times 10^{-5}$ AT $k_* = 0.002/\mathrm{Mpc}$.

(ii) THE LOW ENOUGH VALUE OF $\alpha_{\rm s}$ (IN ORDER TO BE CONSISTENT WITH THE POWER-LAW ΛCDM MODEL)[13]: $|\alpha_{\rm s}| \ll 0.01$. WE DISPLAY CURVES FOR $\alpha_{\rm s} = -0.005$ AND -0.01.

(ii) THE RESOLUTION OF THE HORIZON AND FLATNESS PROBLEMS OF SBB:

$$N_{\rm tot} = N_{\rm HI*} + N_{\rm MI} \simeq \ln\frac{H_0 a_0}{k} + 24.72 + \frac{2}{3}\ln\frac{V_{\rm HI0}^{1/4}}{1\ {\rm GeV}} + \frac{1}{3}\ln\frac{T_{\rm Mrh}}{1\ {\rm GeV}}, \quad \text{AT } k = k_* = 0.002/\mathrm{Mpc}.$$

WE ASSUME THAT $T_{\rm Hrh} < V_{\rm MI0}^{1/4}$ AND, THUS, WE OBTAIN JUST MD DURING THE INTER-INFLATIONARY ERA ($T_{\rm Mrh}$, THE REHEAT TEMPERATURE AFTER MI).

(iii) THE HOMOGENEITY OF THE PRESENT UNIVERSE:

$$s_{\rm i} \gg \delta s_{\rm i}|_{\rm HI} \simeq H_{\rm HI0}/2\pi \quad \text{AND} \quad s_{\rm i} \gg \delta s_{\rm i}|_{\rm MI} \simeq H_{\rm MI0}/2\pi$$

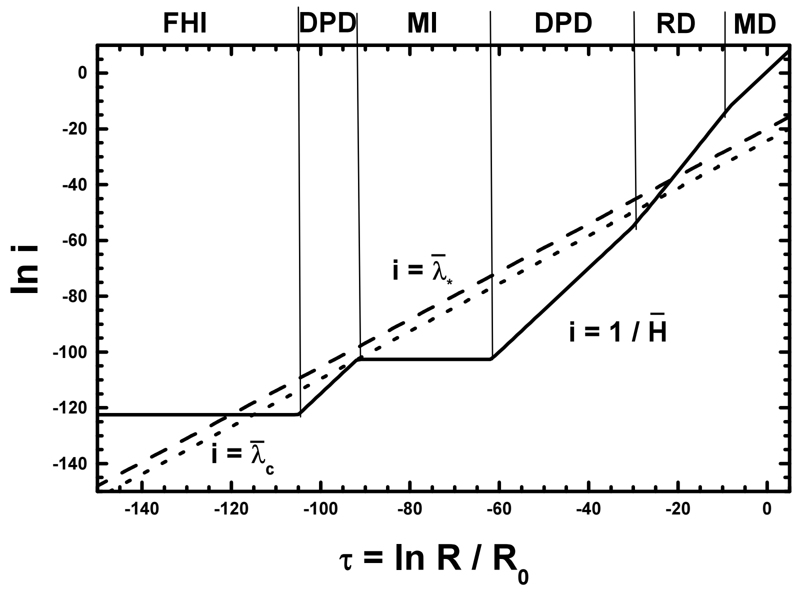
WHERE $H_{\rm HI0} = \sqrt{V_{\rm HI0}}/\sqrt{3}m_{\rm P}$ AND $\delta s_{\rm i}$ ARE THE QUANTUM FLUCTUATIONS OF s DURING FHI.

(iv) THE COMPLETE RANDOMIZATION OF THE STRING AXION AND THE BELONGING OF ALL THE VALUES OF s TO THE RANDOMIZATION REGION WITH EQUAL PROBABILITY:

$$V_{\rm MI0} \lesssim H^4_{\rm HI0} \ \Rightarrow\ s_i/m_{\rm P} \in (0,1).$$

(v) THE NATURALNESS OF MI: $m_s/H_s \lesssim 2.45 \ \Rightarrow\ N_{\rm MI} \gtrsim 0.73\ln(m_{\rm P}/s_{\rm i}) \sim 3.3$ FOR $s_{\rm i}/m_{\rm P} \simeq 0.01$.

[13] *G. Ballesteros, J.A. Casas, and J.R. Espinosa (2006)*



- $\boldsymbol{\tau} = \ln a/a_0 = -\ln(1+z)$: THE LOGARITHMIC TIME,
- $\bar{H} = H/H_0$: THE DIMENSIONLESS HUBBLE PARAMETER,
- $\bar{R}_H = 1/\bar{H}$: THE DIMENSIONLESS PARTICLE HORIZON, $\ln \bar{R}_H \propto 2\boldsymbol{\tau}[1.5\boldsymbol{\tau}]$ FOR RD [MD] ERA AND $\ln \bar{R}_H =$CST. FOR FHI OR MI,
- $\bar{\lambda} = \lambda/a_0$: THE DIMENSIONLESS LENGTH SCALE, $\ln \lambda \propto \boldsymbol{\tau}$,
- $\bar{\lambda}_*[\bar{\lambda}_c]$: THE SCALE CORRESPONDING TO k_* [k_c].

(vi) THE REQUIREMENT THAT THE COSMOLOGICAL SCALES (WITH $k < 0.1/\mathrm{Mpc}$[14]):

- LEAVE THE HORIZON DURING FHI $N_{\mathrm{HI}*} \gtrsim N_{\mathrm{tot}}(k = 0.002/\mathrm{Mpc}) - N_{\mathrm{tot}}(k = 0.1/\mathrm{Mpc}) = 3.9$ AND
- DO NOT RE-ENTER THE HORIZON BEFORE THE ONSET OF MI[15] $N_{\mathrm{HI}*} \gtrsim N_{\mathrm{HIc}}$, WHERE:

$$1 = \frac{k_c}{H_s a_s} = \frac{H_c a_c}{H_s a_{\mathrm{HIf}}}\frac{a_{\mathrm{HIf}}}{a_s} = e^{-N_{\mathrm{HIc}}}\left(\frac{V_{\mathrm{HI0}}}{V_{\mathrm{MI0}}}\right)^{1/2-1/3} \Rightarrow N_{\mathrm{HIc}} = \frac{1}{6}\ln\frac{V_{\mathrm{HI0}}}{V_{\mathrm{MI0}}}$$

THEREFORE, ALL IN ALL WE HAVE: $N_{\mathrm{HI}*} \gtrsim N_{\mathrm{HI}*}^{\mathrm{min}} \simeq 3.9 + \frac{1}{6}\ln\frac{V_{\mathrm{HI0}}}{V_{\mathrm{MI0}}} \sim 10.$

[14] *U. Seljak, A. Slosar, and P. McDonald (2006).*
[15] *C.P. Burgess et al. (2005).*

IV. Numerical Results

We Take $T_{\rm Mrh}=1~{\rm GeV}$ and $m_{3/2}=m_s=1~{\rm TeV}$ Through out. Also, We fix $s_{\rm i}/m_{\rm P}=0.01$ Since (i) $s_{\rm i}<\langle s\rangle\simeq m_{\rm P}$ and so, the Extra Terms of V_{MI0} are Irrelevant (ii) Ensures a Large Available Parameter Space For $n=0.958$ and $v_{_G}\simeq M_{\rm GUT}$ (This Choice Though Signalises a Very Mild Tunning).

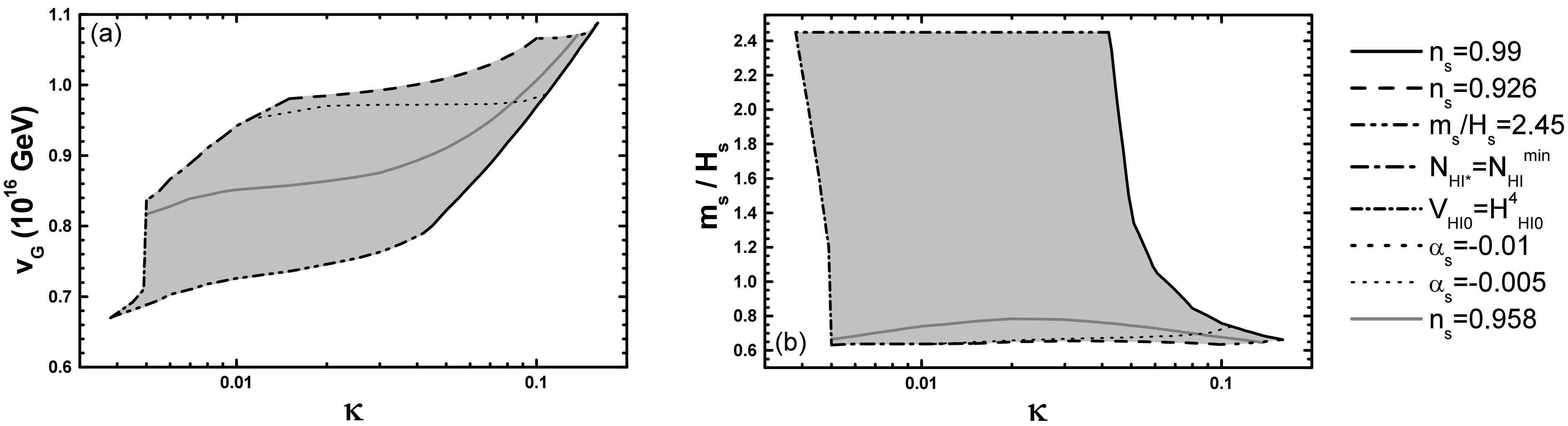
A. Standard-FHI

1. Set-up

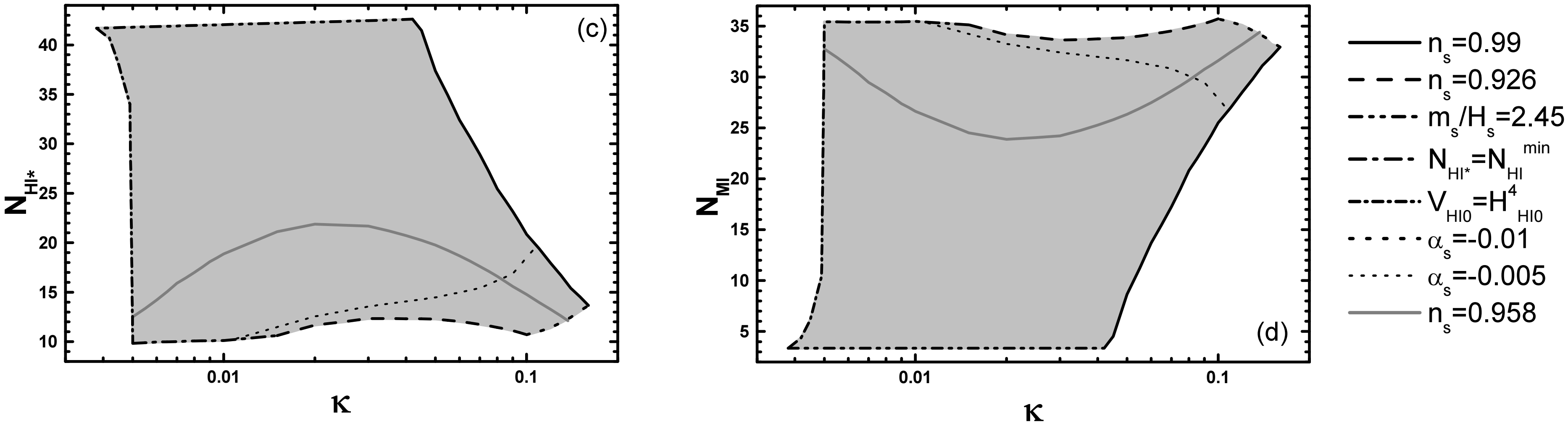
We Consider $G={\rm SU}(3)_{\rm c}\times{\rm SU}(2)_{\rm L}\times{\rm SU}(2)_{\rm R}\times{\rm U}(1)_{\rm B-L}$ (N $=2$).

If $\bar\Phi$ and Φ are ${\rm SU}(2)_{\rm R}$ Doublets With $B-L=-1,~1$ Respectively, no Cosmic Strings Are Produced.

2. Allowed Regions by all the Constraints



For $n_{\rm s}=0.958$, We Obtain $0.004\lesssim\kappa\lesssim0.14,~0.79\lesssim v_{_G}/10^{16}~{\rm GeV}\lesssim1.08,~~-0.002\gtrsim\alpha_{\rm s}\gtrsim-0.01$ and $0.64\lesssim m_s/H_s\lesssim0.77.$



For $n_{\rm s} = 0.958$, We Obtain $10. \lesssim N_{\rm HI*} \lesssim 21.7$ and $35 \gtrsim N_{\rm MI} \gtrsim 24$.

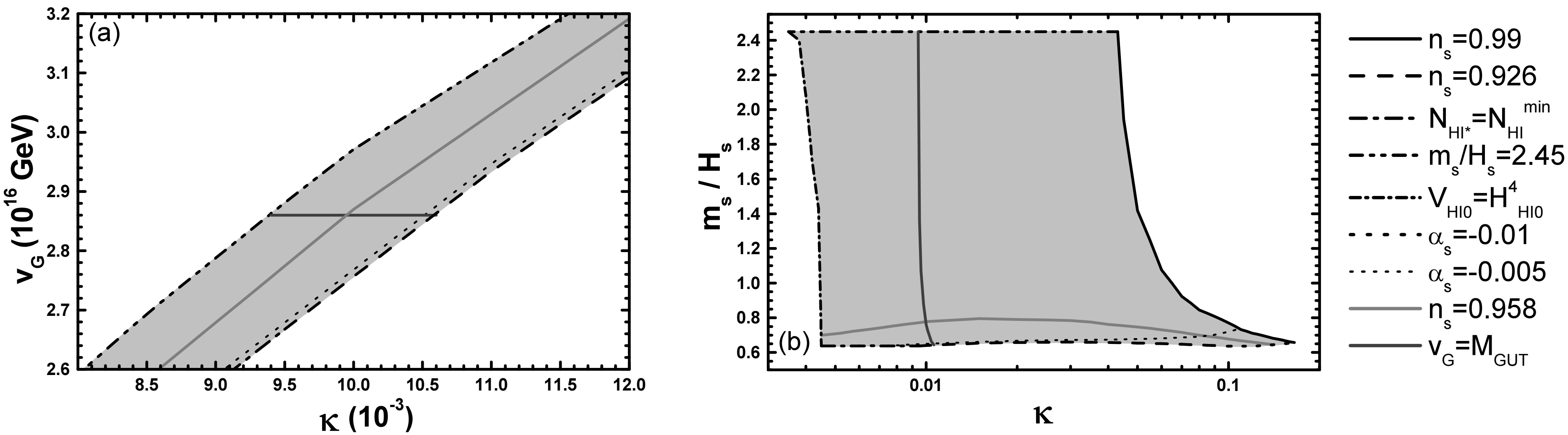
B. Shifted FHI

1. Set-up

We Adopt the Pati-Salam Gauge Group $G = {\rm SU}(4)_{\rm c} \times {\rm SU}(2)_{\rm L} \times {\rm SU}(2)_{\rm R}$ and Fix $M_{\rm S} = 5 \times 10^{17}~{\rm GeV}$.

2. Allowed Regions

The Results are Quite Similar to These for st-FHI (The Bounds on ξ do not cut out any Slice of the Available Parameter Space). But We Can Obtain $v_{_G} = M_{\rm GUT}$.



FOR $n_{\rm s} = 0.958$ AND $v_{_G} = M_{\rm GUT}$, WE OBTAIN $\kappa \simeq 0.01,\ N_{\rm HI*} \simeq 21.,\ |\alpha_{\rm s}| = 0.0018,\ m_s/H_s \simeq 0.77$ AND $N_{\rm MI} \simeq 24.3.$

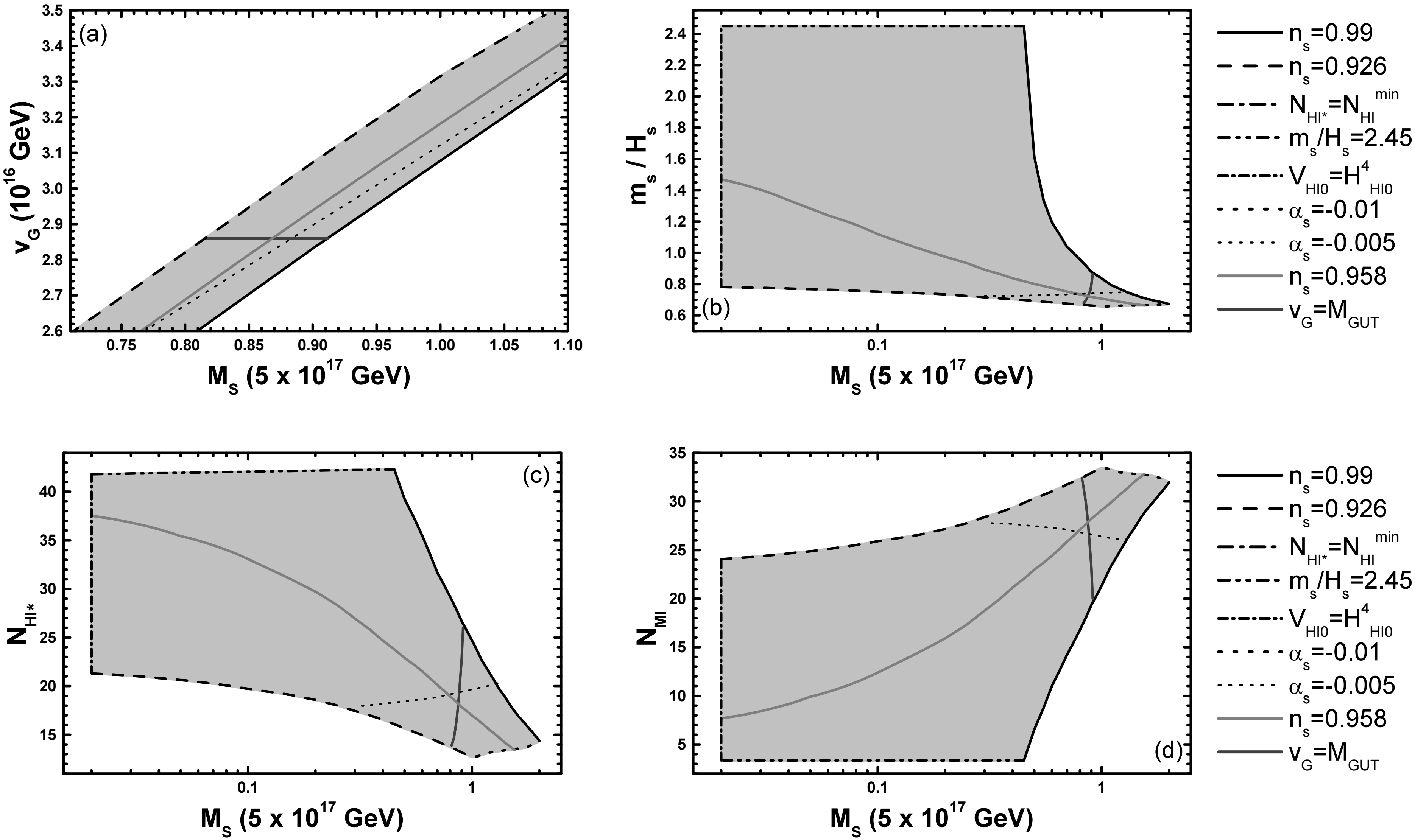
B. SMOOTH FHI

1. SET-UP

WE DO NOT INCLUDE RADIATIVE CORRECTIONS INTO OUR COMPUTATION, AND SO THERE IS NO SPECIFIC GUT.

2. ALLOWED REGIONS

- IN CONTRAST TO ST-FHI AND SH-FHI, $|\alpha_{\rm s}|$ IS CONSIDERABLY ENHANCED IN THE CASE OF SM-FHI FOR $v_{_G} \sim M_{\rm GUT}$
- IN THE CASE OF SM-FHI, SUGRA CORRECTIONS PLAY AN IMPORTANT ROLE FOR EVERY $M_{\rm S}$ IN THE ALLOWED REGION WHEREAS THEY BECOME MORE AND MORE SIGNIFICANT AS κ INCREASES ABOVE 0.01 IN THE CASES OF ST-FHI AND SH-FHI.
- CONTRARY TO ST-FHI AND SH-FHI, THE CONSTRAINT $N_{\rm HI*} \gtrsim N_{\rm HI*}^{\rm min}$ DOES NOT RESTRICT THE PARAMETERS, SINCE IT IS OVERSHADOWED BY THE CONSTRAINT OF $n_{\rm s} \simeq 0.926$.



FOR $n_{\rm s} = 0.958$ AND $v_G = M_{\rm GUT}$, $M_{\rm S}/(5\times10^{17}\,{\rm GeV}) \simeq 0.87$, $N_{\rm HI*} \simeq 18$, $|\alpha_{\rm s}| = 0.005$, $m_s/H_s \simeq 0.72$ AND $N_{\rm MI} \simeq 27.8$.

V. Conclusions

We Presented two-Stage Inflationary Models in Which a Superheavy Scale FHI is Followed by an Intermediate Scale MI. We Confronted These Models With the Data on $P_{\mathcal{R}}$ and $n_{\rm s}$ Within the power-law ΛCDM Model. We Showed That These Restrictions Can be met, Provided That $N_{\rm HI*}$ is Restricted to Rather Low Values (~ 20). For Central Values of $P_{\mathcal{R}}$ and $n_{\rm s}$, We Found That:

$$v_G \begin{cases} < M_{\rm GUT} \text{ and } 10. \lesssim N_{\rm HI*} \lesssim 21.7 & \text{for st-FHI,} \\ = M_{\rm GUT} \text{ and } N_{\rm HI*} \simeq 21 & \text{for sh-FHI,} \\ = M_{\rm GUT} \text{ and } N_{\rm HI*} \simeq 18 & \text{for sm-FHI.} \end{cases}$$

In all cases, MI of the Slow-Roll Type With $m_s/H_s \sim 0.6 - 0.8$ Naturally Produces $N_{\rm MI} = N_{\rm tot} - N_{\rm HI*} \simeq (20 - 30)$. Therefore, MI Complements Successfully FHI.

VI. Future Directions

- What Happens If s Obtains a Mass Term During FHI?
- We can Increase the Reheat Temperature after Complementary Inflation?
- We can Realize a Mechanism of Baryogenesis within this Framework?
- We can Produce the Required $N_{\rm MI} = N_{\rm tot} - N_{\rm HI*} \simeq (20 - 30)$ Through a Thermal Inflation?
- Are There Observational Consequences?