Neutrino mass hierarchy and θ_{13} with a magic baseline beta-beam experiment



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- Beta beam
- India-based Neutrino Observatory (INO)
- Neutrino oscillations with matter effect
- Probing neutrino parameters with a long baseline experiment
- Results
- Conclusions



Beta-beam

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- A pure, intense, collimated beam of ν_e or $\bar{\nu}_e$, essentially background free.
- ν_e or $\bar{\nu}_e$ beams may also be produced at the same time in the set-up
- Origin: beta decay of radioactive ions circulating in a storage ring. No contamination of other types of neutrinos.



Some positive features

- \Rightarrow known energy spectrum
- \Rightarrow high intensity, low systematic errors
- \Rightarrow can be produced with existing CERN facilities or planned upgrades.



Beta Beam (contd.)

- The ν_e ($\bar{\nu}_e$) beams are produced via the β decay of accelerated and completely ionized ${}^{18}Ne$ (${}^{6}He$) ions.
- ${}^{18}_{10}Ne \rightarrow {}^{18}_{9}F + e^+ + \nu_e$
- ${}^6_2He \rightarrow {}^6_3Li + e^- + \bar{\nu}_e$
- Both beams can run simultaneously in the storage ring. The boost factors are fixed by the ratio e/m: $\gamma ({}^{18}Ne) = 1.67 \cdot \gamma ({}^{6}He)$
- The number of injected ions in case of anti-neutrinos can be 2.9×10^{18} /year and for neutrinos 1.1×10^{18} /year.
- The $\nu_e/\bar{\nu}_e$ flux is readily obtained from standard beta decay.



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lon	τ (S)	E_0 (MeV)	f	Decay fraction	Beam
$^{18}_{10}$ Ne	2.41	3.41	393.5	92.1%	$ u_e$
6_2 He	1.17	3.51	462.6	100%	$ar{ u}_e$
8_5 B	1.11	13.92	501543.0	100%	$ u_e$
$\frac{8}{3}$ Li	1.20	12.96	350500.5	100%	$ar{ u}_e$

Comparison of different source ions

Larger end-point energy, E_0 , is preferred



ν_e Spectrum



Boosted on-axis spectrum of neutrinos at the far detector assuming no oscillation.



$\bar{\nu}_e$ Spectrum



Boosted on-axis spectrum of anti-neutrinos at the far detector assuming no oscillation.



Neutrino Factory/Beta Beam

	ν -factory	eta-be	eam
Beam	$\mu^- \to \nu_\mu + \bar{\nu}_e + e^-$	${}^8_3Li \to \bar{\nu}_e$	
	$\mu^+ \to \bar{\nu}_\mu + \nu_e + e^+$	${}^8_5B \to \nu_e$	
No./yr	10 ²⁰	$\sim 10^{18}$	
Energy	$E_{\mu} = 20 - 50$	γ = 250	γ = 500
(GeV)		$E_{\nu,\bar{\nu}}\sim3$	$E_{\nu,\bar{\nu}}\sim 6$

Beta beam γ : \leq 250 (SPS) 250-600 (super-SPS)

High $\gamma \Rightarrow$ higher energy, better collimation, longer baseline



The India-based Neutrino Observatory (INO)

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INO

- \Rightarrow The India-based Neutrino Observatory (INO)
- $\Rightarrow\,$ Earlier experiments: KGF proton decay, atmospheric ν_{μ} detection
- ⇒ ICAL: a magnetized Iron calorimeter with interleaved Glass RPC detectors
- \Rightarrow good efficiency of charge identification (\sim 95%)
- \Rightarrow Excellent energy determination for μ^{\pm} with $E \geq 1$ GeV
- $\Rightarrow \nu_e \rightarrow \nu_\mu$ oscillation signal \Rightarrow muon track



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Schematic plan of the 50 kTon ICAL detector for INO.



- \Rightarrow Two possible locations
 - (a) Singara (PUSHEP) in the Nilgiris

(b) Rammam in the Darjeeling Himalayas (L = 6937 km from CERN)

- ⇒ PUSHEP chosen ~ 250km from Bangalore (L = 7152 km from CERN)
- \Rightarrow A (50+50) kton Iron detector
- \Rightarrow Funding considerations in final stage



INO (First phase)

- Atmospheric neutrino oscillation studies
- Effi cient charge identifi cation \Rightarrow Permits good discrimination between ν_{μ} and $\bar{\nu}_{\mu}$ events
- Simulation: (up/down) vs. (L/E) exhibits oscillatory dip
- Precision measurement of Δm^2_{23}
- Determination of its sign
- Simulation, prototype construction, ... in progress
- R & D support, Target date: 2012
- International collaborations sought



Three-favour oscillations

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Three-flavour oscillations

- ⇒ Neutrino parameters: neutrino mass eigenvalues and the PMNS mixing matrix
- ⇒ neutrino flavour states $|\nu_{\alpha}\rangle$ ($\alpha = e, \mu, \tau$) are linear superpositions of the mass eigenstates $|\nu_i\rangle$ (*i* = 1, 2, 3) with masses m_i

$$|\nu_{\alpha}\rangle = \sum_{i} U_{\alpha i} |\nu_{i}\rangle$$

 \Rightarrow $U \equiv 3 \times 3$ unitary matrix (PMNS) parametrized as:

$$U = V_{23} W_{13} V_{12}$$



Three-flavours (contd.)

where

$$V_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, W_{13} = \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix},$$
$$V_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}.$$

 $\Rightarrow c_{12} = \cos \theta_{12}, s_{12} = \sin \theta_{12}$ etc.

 $\Rightarrow \delta$ denotes the CP-violating (Dirac) phase

(Majorana phases ignored)



Three-flavours (contd.)

The probability that an initial ν_f of energy *E* gets converted to a ν_g after traveling a distance *L* in vacuum

$$P(\nu_f \to \nu_g) = \delta_{fg} - 4 \sum_{j>i} \operatorname{Re}(U_{fi}^* U_{gi} U_{fj} U_{gj}^*) \sin^2(1.27\Delta m_{ij}^2 \frac{L}{E})$$

$$\pm 2 \sum_{j>i} \operatorname{Im}(U_{fi}^* U_{gi} U_{fj} U_{gj}^*) \sin(2.54\Delta m_{ij}^2 \frac{L}{E})$$

L is expressed in km, *E* in GeV and Δm^2 in eV². The – (+) refers to neutrinos (anti-neutrinos). Т



Matter effects

Probabilites in matter

 \Rightarrow Interactions in matter modify the oscillation probability



Probabilites in matter

- \Rightarrow Interactions in matter modify the oscillation probability
- \Rightarrow the 3-flavour neutrino evolution equation in matter :

$$\begin{split} i\frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} &= \\ \begin{bmatrix} \frac{1}{2E}U \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} U^{\dagger} + \begin{pmatrix} V_{CC} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{bmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \end{split}$$

- $V_{CC} = \sqrt{2}G_F n_e$ (matter-induced potential)
- n_e is the electron number density



- ⇒ Atmospheric neutrinos 3σ : $2.0 \times 10^{-3} \text{eV}^2 < |\Delta_{32}| < 3.2 \times 10^{-3} \text{eV}^2$ and $\sin^2 2\theta_{23} > 0.9$
- $\Rightarrow \text{ solar neutrinos } 3\sigma : 0.25 < \sin^2 \theta_{12} < 0.39 ,$ $7.2 \times 10^{-5} \text{eV}^2 < \Delta_{12} < 9.2 \times 10^{-5} \text{eV}^2$
- ⇒ current bound on CHOOZ mixing angle θ_{13} from the global oscillation analysis : $\sin^2 2\theta_{13} < 0.17$
- ⇒ two large mixing angles and the relative oscillation frequencies open the possibility to test CP violation in the neutrino sector, if θ_{13} and δ are not vanishingly small



Neutrino mixing

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Neutrino mixing (contd.)

Unsolved issues

- \Rightarrow The sign of Δm_{31}^2 is not known. Neutrino mass spectrum can be direct or inverted hierarchical
- \Rightarrow Only an upper limit on θ_{13}
- \Rightarrow The CP phase, δ , is unconstrained

Eightfold problem of parameter degeneracies:

- \Rightarrow the (θ_{13}, δ_{CP}) intrinsic degeneracy
- \Rightarrow the $(sgn(\Delta m_{31}^2), \delta_{CP})$ degeneracy
- \Rightarrow the ($\theta_{23}, \pi/2 \theta_{23}$) degeneracy



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Magic baseline

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Magic baseline

The appearance probability ($\nu_e \rightarrow \nu_\mu$) in matter, upto second order in the small parameters $\alpha \equiv \Delta m_{12}^2 / \Delta m_{13}^2$ and $\sin 2\theta_{13}$,

$$P_{e\mu} \simeq \sin^2 2\theta_{13} \sin^2 \theta_{23} \frac{\sin^2[(1-\hat{A})\Delta]}{(1-\hat{A})^2}$$

$$\pm \alpha \sin 2\theta_{13} \xi \sin \delta \sin(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1-\hat{A})\Delta]}{(1-\hat{A})}$$

$$+ \alpha \sin 2\theta_{13} \xi \cos \delta \cos(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1-\hat{A})\Delta]}{(1-\hat{A})}$$

$$+ \alpha^2 \cos^2 \theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2};$$

where $\Delta \equiv \Delta m_{13}^2 L/(4E)$, $\xi \equiv \cos \theta_{13} \sin 2\theta_{12} \sin 2\theta_{23}$, and $\hat{A} \equiv \pm (2\sqrt{2}G_F n_e E)/\Delta m_{13}^2$.



Magic Baseline (contd.)

If one chooses: $\sin(\hat{A}\Delta) = 0$

- The δ dependence disappears from $P(\nu_e \rightarrow \nu_\mu)$.
- A clean measurement of the hierarchy and θ_{13} is possible without any correlation with δ .



Magic Baseline (contd.)

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The first non-trivial solution: $\sqrt{2}G_F n_e L = 2\pi$ (indep of *E*)

- Isoscalar medium of constant density ρ : $L_{\text{magic}}[\text{km}] \approx 32726/\rho[\text{gm/cm}^3]$.
- The averaged density for the CERN-INO path is $\rho \simeq$ 4.25 gm/cc $\Rightarrow L_{magic} \simeq$ 7690 km.



- The longer baseline captures a matter-induced contribution to the neutrino parameters, essential for probing the sign of Δm^2_{23} .
- The CERN-INO baseline, 7152 km, close to the 'magic' value, ensures essentially no dependence of the final results on δ .
- This permits a clean measurement of θ_{13} avoiding the degeneracy issues which plague other baselines.



- The very long CERN-INO baseline provides an excellent avenue to pin-down matter-induced contributions
- In particular, a resonance occurs at

$$E_{res} \equiv \frac{|\Delta m_{31}^2|\cos 2\theta_{13}}{2\sqrt{2}G_F N_e},$$

• For $|\Delta m_{31}^2| = 2.5 \times 10^{-3} \text{ eV}^2$, $\sin^2 2\theta_{13} = 0.1$ and the PREM profile $\rho_{av} = 4.13$ gm/cc, it is $E_{res} \simeq 6.1$ GeV.



Transition Probability $P_{e\mu}$



Transition probability for different baselines. Normal vs. Inverted hierarchy.





 θ_{13} variation.



Results

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Detector assumptions

Total Mass	50 kton
Energy threshold	1.5 GeV
Detection Efficiency (ϵ)	60%
Charge Identification Efficiency (f_{ID})	95%



No. of events



Event rates.



No. of events (contd.)



Sensitivity to matter profile



Degeneracy



Iso-event curves: dependence on δ_{CP} .



The χ^2 function

Assume Poissonian distribution and define

$$\chi^2(\{\omega\}) = \min_{\xi_k} \left[2\left(\tilde{N}^{th} - N^{ex} - N^{ex} \ln \frac{\tilde{N}^{th}}{N^{ex}}\right) + \sum_k \xi_k^2 \right]$$

 $\{\omega\}$: oscillation parameters, $\{\xi_k\}$: "pulls", where k: runs over systematic uncertainties

$$\tilde{N}^{th}(\{\omega\},\{\xi_k\}) = N^{th}(\{\omega\}) \left[1 + \sum_{k=1}^{K} \pi^k \xi_k\right] + \mathcal{O}(\xi_k^2) ,$$

Minimise χ^2 by varying over $\{\omega\}$ and finally marginalise over Δm_{31}^2 , $\sin^2 2\theta_{23}$ by minimising $\chi^2_{total} = \chi^2 + \chi^2_{prior}$



Neutrino mass ordering



The minimum value of $\sin^2 2\theta_{13}$ as a function of the boost γ at which the wrong hierarchy can be disfavored at the 90% and 3σ C.L. For ν_e ($\overline{\nu_e}$) true hierarchy is assumed normal (inverted)



θ_{13} sensitivity



 $\sin^2 2\theta_{13}$ limit below which experiment is insensitive



θ_{13} measurement



 $\sin^2 2\theta$ measurement. 5 years, neutrino channel, normal hierarchy. "measured" $\sin^2 2\theta_{13}$ (upper), corresponding precision (lower)



- Beta-beam source at CERN and magnetised iron calorimeter at INO: Good for exploring θ_{13} and sign(Δm_{23}^2)
- The baseline is close to the "magic" value and hence avoids degeneracy problems
- Large distance captures significant matter effect
- Neutrino enenrgy for boost $\gamma \simeq$ 500 gives resonant enhancement
- Results are very encouraging

