Neutrino mass hierarchy and θ_{13} with a magic baseline beta-beam experiment

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Plan

- Beta beam
- India-based Neutrino Observatory (INO)
- Neutrino oscillations with matter effect
- Probing neutrino parameters with a long baseline experiment
- Results
- Conclusions

Beta-beam

Beta Beam

- A pure, intense, collimated beam of ν_e or $\bar{\nu}_e$, essentially background free.

- ν_e or $\bar{\nu}_e$ beams may also be produced at the same time in the set-up

- Origin: beta decay of radioactive ions circulating in a storage ring. No contamination of other types of neutrinos.

Some positive features

$\Rightarrow$ known energy spectrum

$\Rightarrow$ high intensity, low systematic errors

$\Rightarrow$ High Lorentz boost of the parent ions $\Rightarrow$ better collimation and higher energy of beam

$\Rightarrow$ can be produced with existing CERN facilities or planned upgrades.

Beta Beam (contd.)

- The ν_e ($\bar{\nu}_e$) beams are produced via the β decay of accelerated and completely ionized ^{18}Ne (^{6}He) ions.
- $^{18}_{10}Ne \rightarrow {}^{18}_{9}F + e^{+} + \nu_e$
- $^{6}_{2}He \rightarrow {}^{6}_{3}Li + e^{-} + \bar{\nu}_e$
- Both beams can run simultaneously in the storage ring. The boost factors are fixed by the ratio e/m:
  $\gamma\left({}^{18}Ne\right) = 1.67 \cdot \gamma\left({}^{6}He\right)$
- The number of injected ions in case of anti-neutrinos can be 2.9×10^{18}/year and for neutrinos 1.1×10^{18}/year.
- The $\nu_e/\bar{\nu}_e$ flux is readily obtained from standard beta decay.

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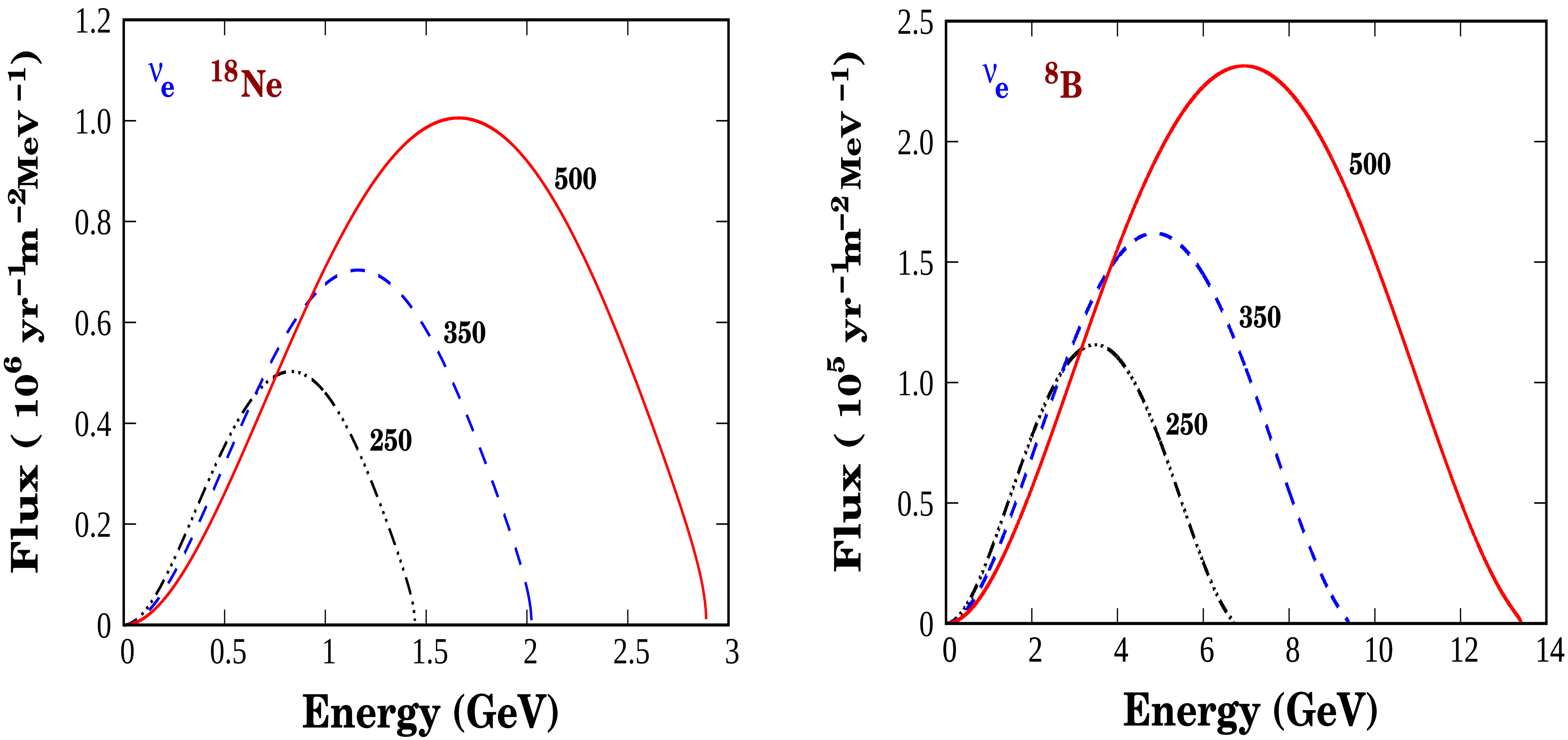
Beta Beam: Ion soures

Ion	τ (s)	E_0 (MeV)	f	Decay fraction	Beam
${}^{18}_{10}$Ne	2.41	3.41	393.5	92.1%	ν_e
${}^{6}_{2}$He	1.17	3.51	462.6	100%	$\bar{\nu}_e$
${}^{8}_{5}$B	1.11	13.92	501543.0	100%	ν_e
${}^{8}_{3}$Li	1.20	12.96	350500.5	100%	$\bar{\nu}_e$

Comparison of different source ions

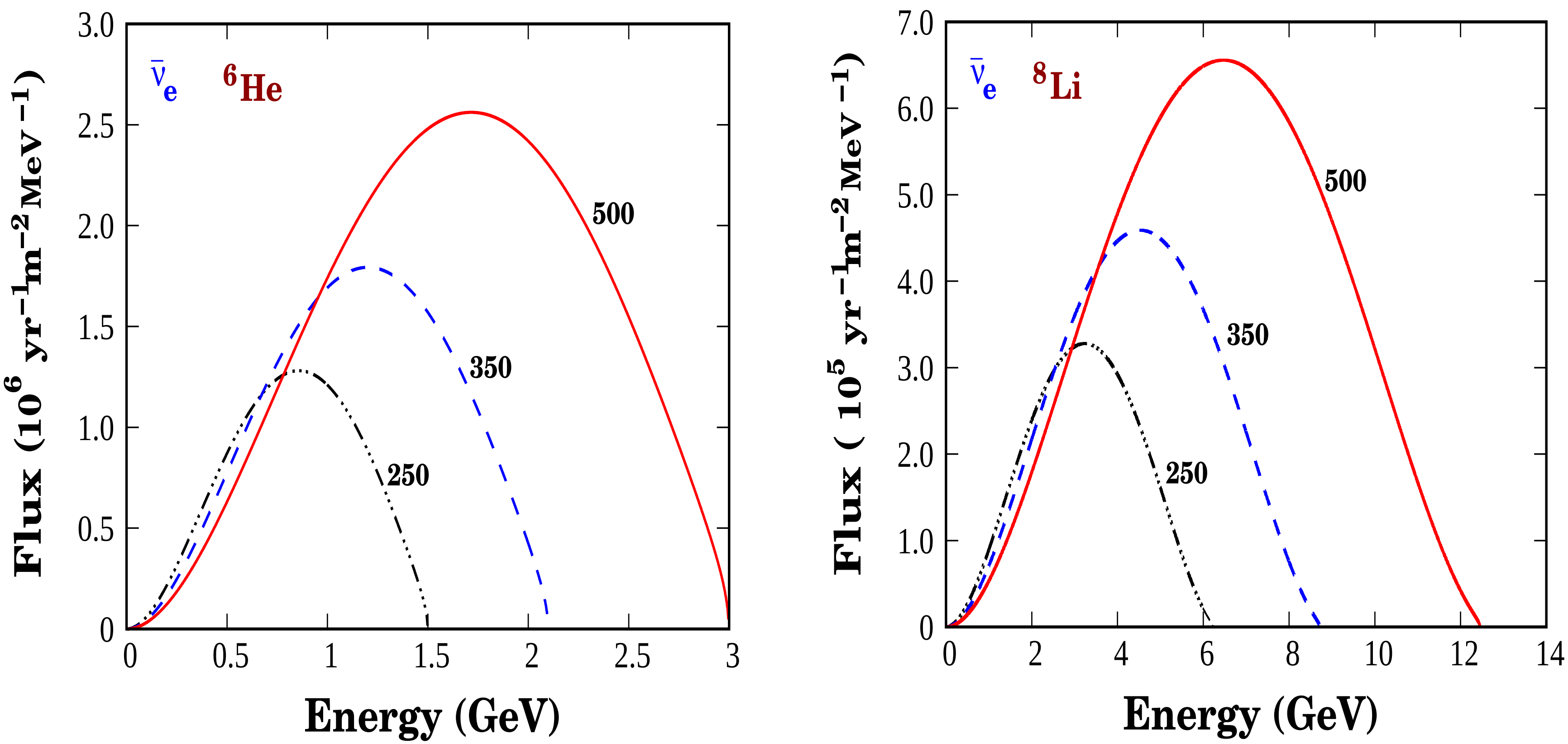
Larger end-point energy, E_0, is preferred

ν_e Spectrum



Boosted on-axis spectrum of neutrinos at the far detector assuming no oscillation.

$\bar{\nu}_e$ Spectrum



Boosted on-axis spectrum of anti-neutrinos at the far detector assuming no oscillation.

Neutrino Factory/Beta Beam

	ν-factory	β-beam	
Beam	$\mu^- \rightarrow \nu_\mu + \bar{\nu}_e + e^-$ $\mu^+ \rightarrow \bar{\nu}_\mu + \nu_e + e^+$	${}^8_3Li \rightarrow \bar{\nu}_e$ ${}^8_5B \rightarrow \nu_e$	
No./yr	10^{20}	$\sim 10^{18}$	
Energy (GeV)	E_μ = 20 – 50	γ = 250 $E_{\nu,\bar{\nu}} \sim 3$	γ = 500 $E_{\nu,\bar{\nu}} \sim 6$

Beta beam γ : $\leq$ 250 (SPS)
250-600 (super-SPS)

High $\gamma \Rightarrow$ higher energy, better collimation, longer baseline

The India-based Neutrino Observatory (INO)

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⇒ Earlier experiments: KGF proton decay, atmospheric ν_μ detection

⇒ ICAL: a magnetized Iron calorimeter with interleaved Glass RPC detectors

⇒ good effi ciency of charge identifi cation ($\sim$ 95%)

⇒ Excellent energy determination for $\mu^\pm$ with $E \geq$ 1 GeV

⇒ $\nu_e \rightarrow \nu_\mu$ oscillation signal ⇒ muon track

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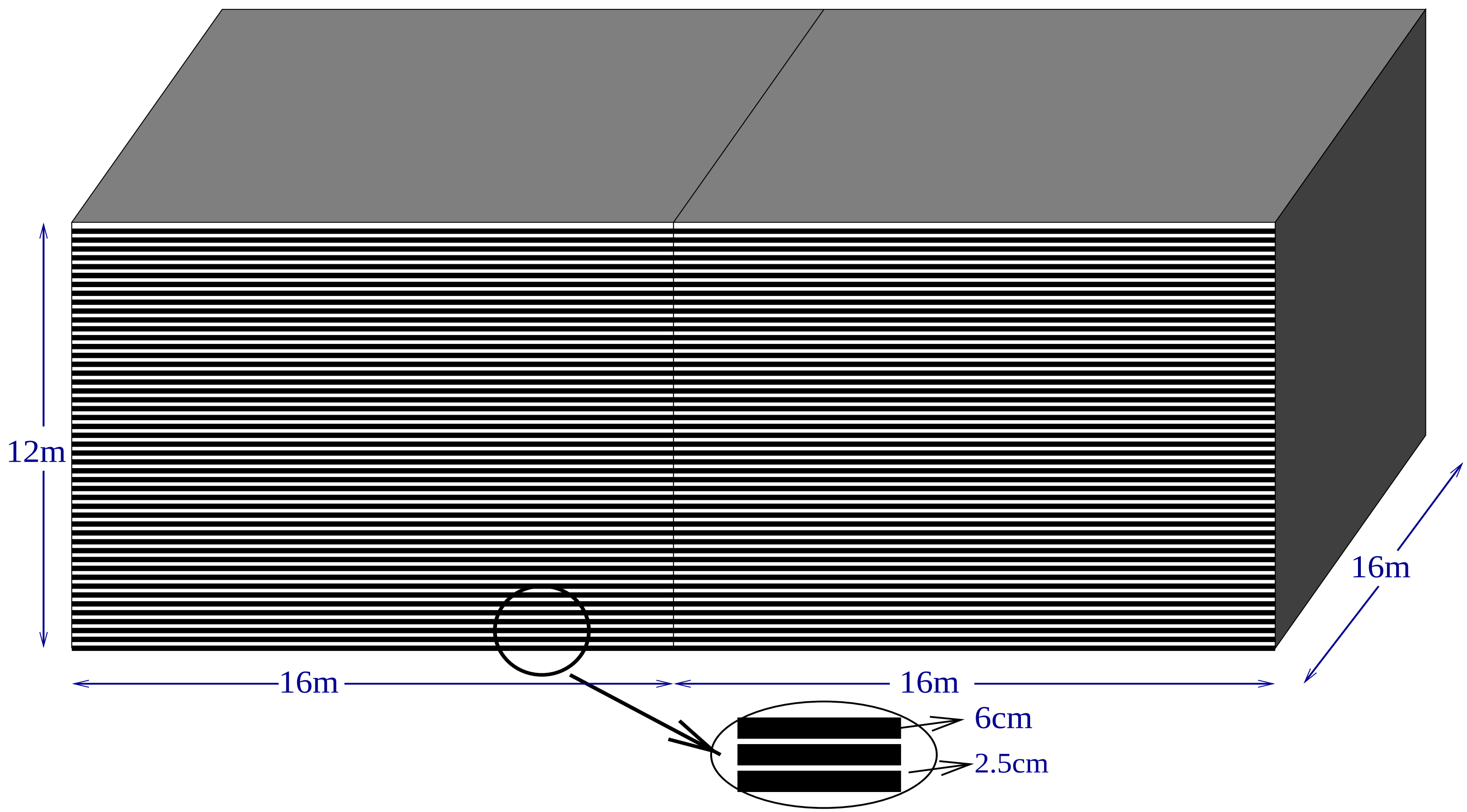

Schematic plan of the 50 kTon ICAL detector for INO.

INO

$\Rightarrow$ Two possible locations

(a) Singara (PUSHEP) in the Nilgiris

(b) Rammam in the Darjeeling Himalayas ($L = 6937$ km from CERN)

$\Rightarrow$ PUSHEP chosen $\sim$ 250km from Bangalore ($L = 7152$ km from CERN)

$\Rightarrow$ A (50+50) kton Iron detector

$\Rightarrow$ Funding considerations in fi nal stage

INO (First phase)

- Atmospheric neutrino oscillation studies
- Efficient charge identification $\Rightarrow$ Permits good discrimination between ν_μ and $\bar{\nu}_\mu$ events
- Simulation: (up/down) vs. (L/E) exhibits oscillatory dip
- Precision measurement of Δm^2_{23}
- Determination of its sign
- Simulation, prototype construction, ... in progress
- R & D support, Target date: 2012
- International collaborations sought

Three-flavour oscillations

Three-flavour oscillations

- $\Rightarrow$ Neutrino parameters: neutrino mass eigenvalues and the PMNS mixing matrix

- $\Rightarrow$ neutrino flavour states $|\nu_\alpha\rangle$ ($\alpha = e, \mu, \tau$) are linear superpositions of the mass eigenstates $|\nu_i\rangle$ (i = 1, 2, 3) with masses m_i

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i} |\nu_i\rangle$$

- $\Rightarrow$ $U \equiv 3 \times 3$ unitary matrix (PMNS) parametrized as:

$$U = V_{23} W_{13} V_{12}$$

Three-flavours (contd.)

where

$$V_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, W_{13} = \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix},$$

$$V_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}.$$

$\Rightarrow$ $c_{12} = \cos\theta_{12}$, $s_{12} = \sin\theta_{12}$ etc.

$\Rightarrow$ δ denotes the CP-violating (Dirac) phase

(Majorana phases ignored)

Three-flavours (contd.)

The probability that an initial ν_f of energy E gets converted to a ν_g after traveling a distance L in vacuum

$$P(\nu_f \to \nu_g) = \delta_{fg} \quad - \quad 4\sum_{j>i} \mathrm{Re}(U^*_{fi}U_{gi}U_{fj}U^*_{gj})\sin^2(1.27\Delta m^2_{ij}\frac{L}{E})$$

$$\pm \quad 2\sum_{j>i} \mathrm{Im}(U^*_{fi}U_{gi}U_{fj}U^*_{gj})\sin(2.54\Delta m^2_{ij}\frac{L}{E})$$

L is expressed in km, E in GeV and Δm^2 in eV^2.

The – (+) refers to neutrinos (anti-neutrinos).

Matter effects

Probabilites in matter

$\Rightarrow$ Interactions in matter modify the oscillation probability

Matter effects

Probabilites in matter

⇒ Interactions in matter modify the oscillation probability

⇒ the 3-flavour neutrino evolution equation in matter :

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} =$$

$$\left[\frac{1}{2E}U\begin{pmatrix}m_1^2 & 0 & 0\\ 0 & m_2^2 & 0\\ 0 & 0 & m_3^2\end{pmatrix}U^\dagger + \begin{pmatrix}V_{CC} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right]\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$$

- $V_{CC} = \sqrt{2}G_F n_e$ (matter-induced potential)
- n_e is the electron number density

Neutrino mixing

⇒ Atmospheric neutrinos 3σ :
$2.0 \times 10^{-3}\text{eV}^2 < |\Delta_{32}| < 3.2 \times 10^{-3}\text{eV}^2$ and $\sin^2 2\theta_{23} > 0.9$

⇒ solar neutrinos 3σ : $0.25 < \sin^2\theta_{12} < 0.39$,
$7.2 \times 10^{-5}\text{eV}^2 < \Delta_{12} < 9.2 \times 10^{-5}\text{eV}^2$

⇒ current bound on CHOOZ mixing angle θ_{13} from the global oscillation analysis : $\sin^2 2\theta_{13} < 0.17$

⇒ two large mixing angles and the relative oscillation frequencies open the possibility to test CP violation in the neutrino sector, if θ_{13} and δ are not vanishingly small

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Neutrino mixing (contd.)

Unsolved issues

- ⇒ The sign of Δm^2_{31} is not known. Neutrino mass spectrum can be direct or inverted hierarchical
- ⇒ Only an upper limit on θ_{13}
- ⇒ The CP phase, δ, is unconstrained

Eightfold problem of parameter degeneracies:

- ⇒ the $(\theta_{13}, \delta_{CP})$ intrinsic degeneracy
- ⇒ the $(sgn(\Delta m^2_{31}), \delta_{CP})$ degeneracy
- ⇒ the $(\theta_{23}, \pi/2 - \theta_{23})$ degeneracy

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Magic baseline

Magic baseline

The appearance probability ($\nu_e \to \nu_\mu$) in matter, upto second order in the small parameters $\alpha \equiv \Delta m^2_{12}/\Delta m^2_{13}$ and $\sin 2\theta_{13}$,

$$
\begin{aligned}
P_{e\mu} &\simeq \sin^2 2\theta_{13}\, \sin^2\theta_{23} \frac{\sin^2[(1-\hat{A})\Delta]}{(1-\hat{A})^2} \\
&\pm \alpha \sin 2\theta_{13}\, \xi \sin\delta \sin(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1-\hat{A})\Delta]}{(1-\hat{A})} \\
&+ \alpha \sin 2\theta_{13}\, \xi \cos\delta \cos(\Delta) \frac{\sin(\hat{A}\Delta)}{\hat{A}} \frac{\sin[(1-\hat{A})\Delta]}{(1-\hat{A})} \\
&+ \alpha^2 \cos^2\theta_{23} \sin^2 2\theta_{12} \frac{\sin^2(\hat{A}\Delta)}{\hat{A}^2};
\end{aligned}
$$

where $\Delta \equiv \Delta m^2_{13} L/(4E)$, $\xi \equiv \cos\theta_{13}\, \sin 2\theta_{12}\, \sin 2\theta_{23}$,
and $\hat{A} \equiv \pm(2\sqrt{2} G_F n_e E)/\Delta m^2_{13}$.

Magic Baseline (contd.)

If one chooses: $\sin(\hat{A}\Delta) = 0$

- The δ dependence disappears from $P(\nu_e \rightarrow \nu_\mu)$.
- A clean measurement of the hierarchy and θ_{13} is possible without any correlation with δ.

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The first non-trivial solution: $\sqrt{2} G_F n_e L = 2\pi$ (indep of E)

- Isoscalar medium of constant density ρ : $L_{\rm magic}[\rm km] \approx 32726/\rho[\rm gm/cm^3]$.
- The averaged density for the CERN-INO path is $\rho \simeq$ 4.25 gm/cc $\Rightarrow L_{\rm magic} \simeq$ 7690 km.

CERN – INO long baseline

- The longer baseline captures a matter-induced contribution to the neutrino parameters, essential for probing the sign of Δm^2_{23}.
- The CERN-INO baseline, 7152 km, close to the 'magic' value, ensures essentially no dependence of the final results on δ.
- This permits a clean measurement of θ_{13} avoiding the degeneracy issues which plague other baselines.

Resonance in matter effect

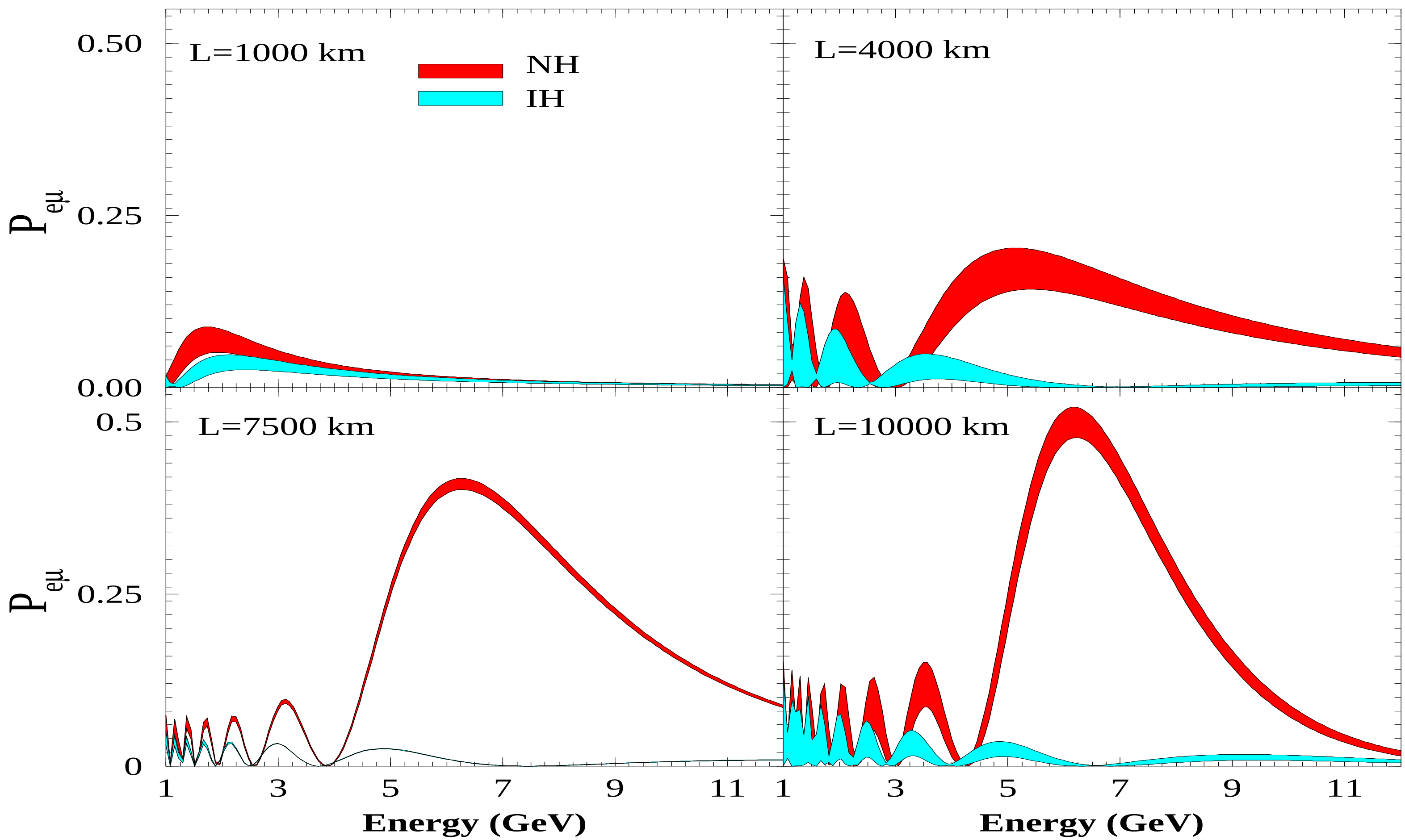
- The very long CERN-INO baseline provides an excellent avenue to pin-down matter-induced contributions

- In particular, a resonance occurs at

$$E_{res} \equiv \frac{|\Delta m^2_{31}| \cos 2\theta_{13}}{2\sqrt{2} G_F N_e},$$

- For $|\Delta m^2_{31}| = 2.5 \times 10^{-3}$ eV^2, $\sin^2 2\theta_{13} = 0.1$ and the PREM profile $\rho_{av} = 4.13$ gm/cc, it is $E_{res} \simeq 6.1$ GeV.

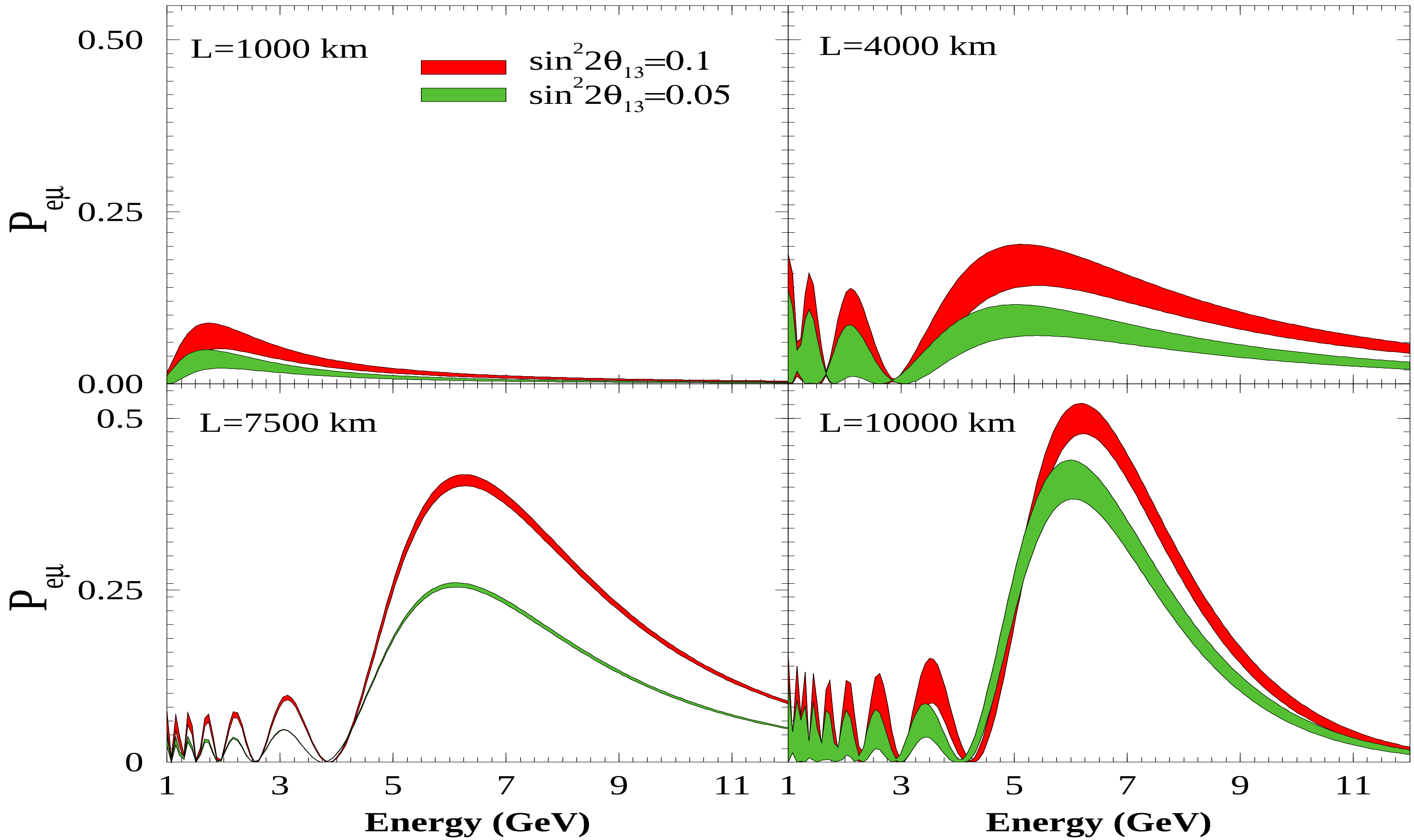
Transition Probability $P_{e\mu}$



Transition probability for different baselines.

Normal vs. Inverted hierarchy.

Transition Probability $P_{e\mu}$



Transition probability for different baselines.

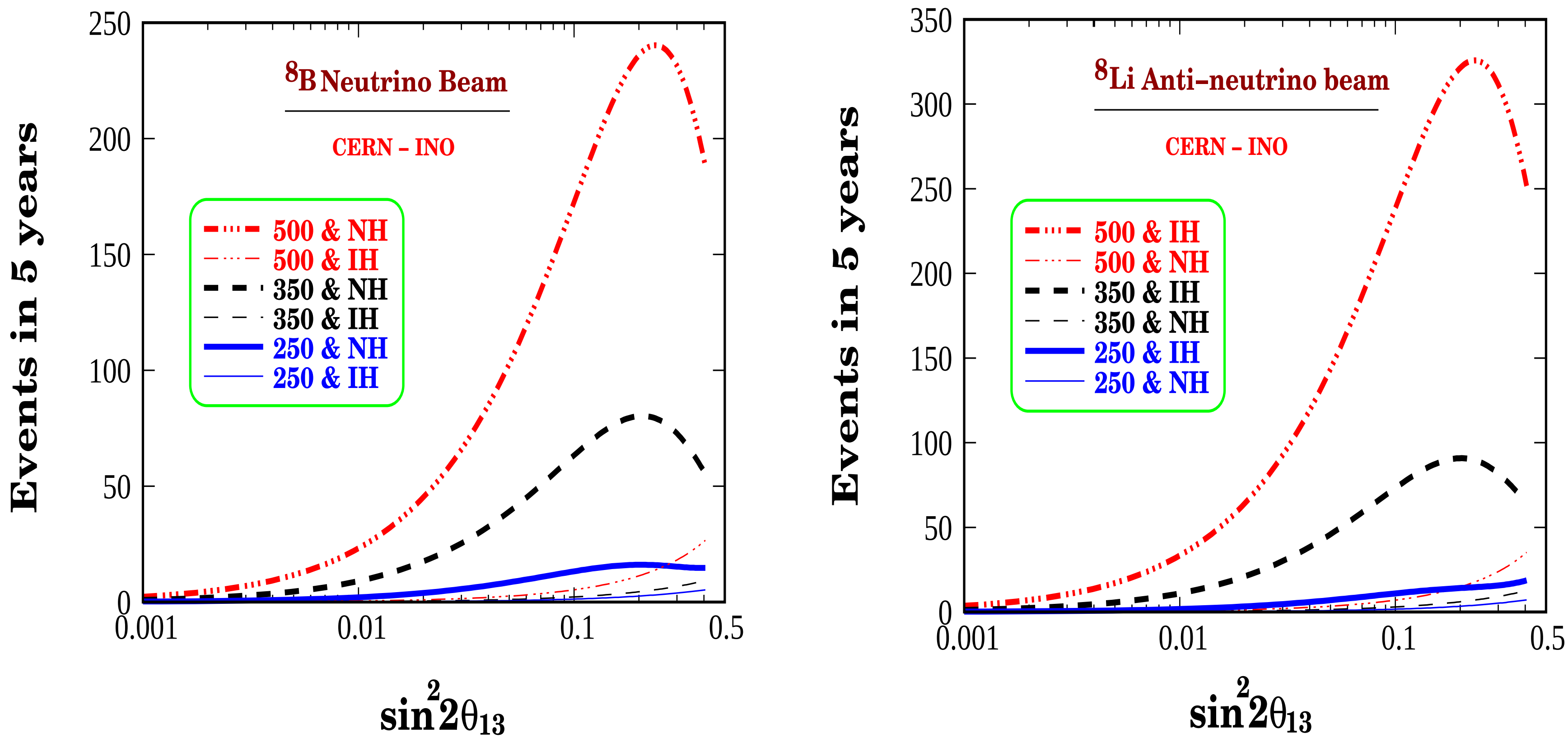
θ_{13} variation.

Results

Detector assumptions

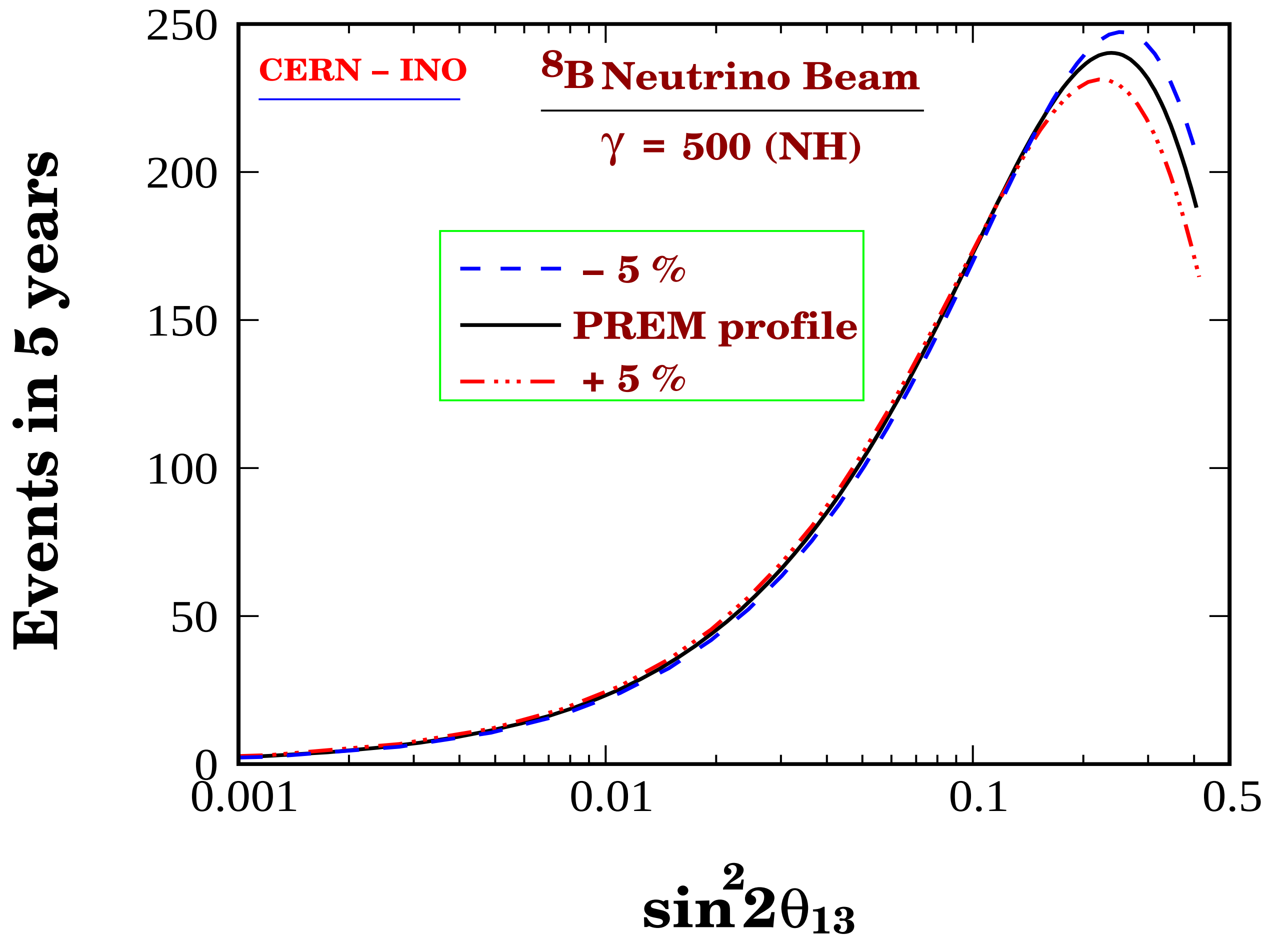
Total Mass	50 kton
Energy threshold	1.5 GeV
Detection Efficiency (ϵ)	60%
Charge Identification Efficiency (f_{ID})	95%

No. of events



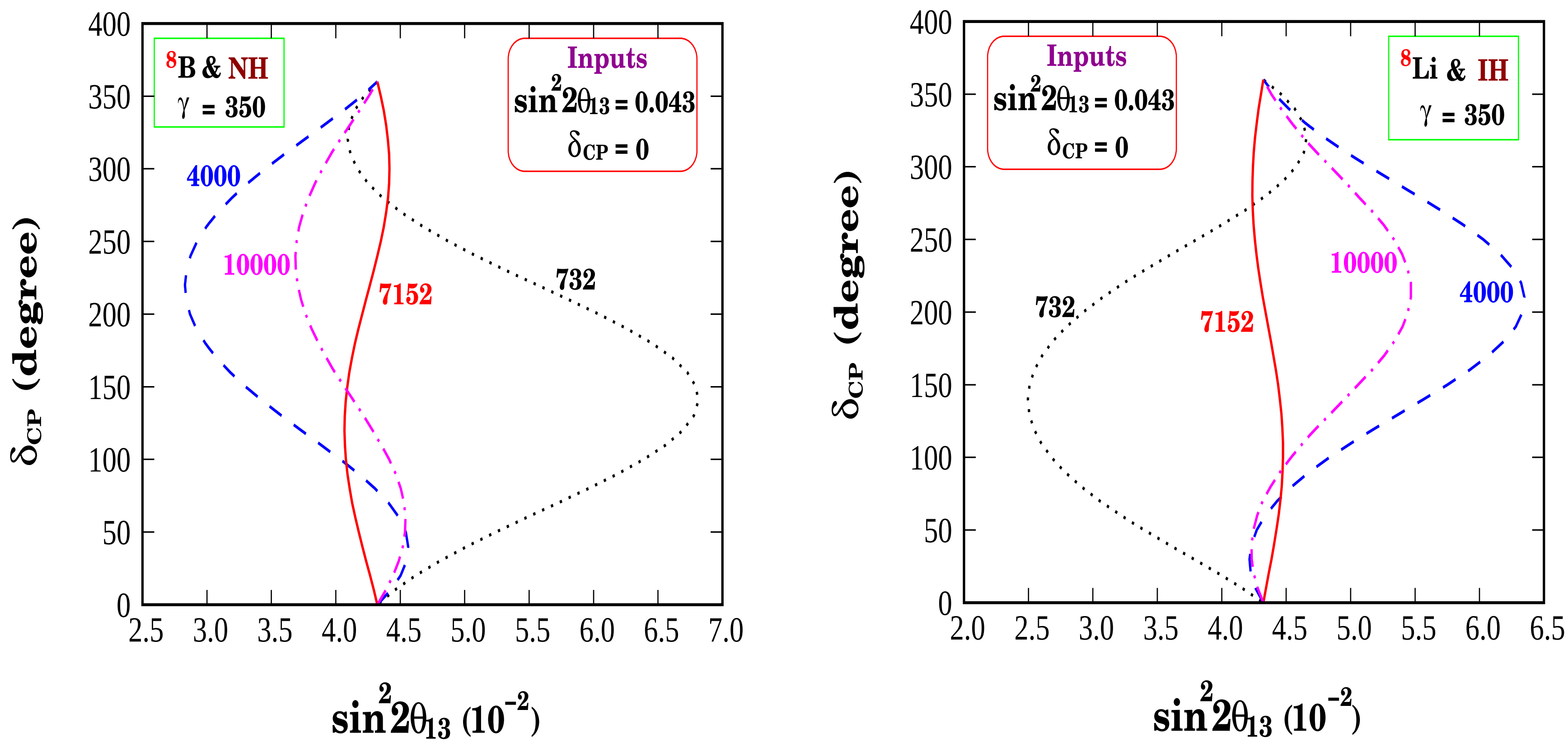
Event rates.

No. of events (contd.)



Sensitivity to matter profi le

Degeneracy



Iso-event curves: dependence on δ_{CP}.

The χ^2 function

Assume Poissonian distribution and define

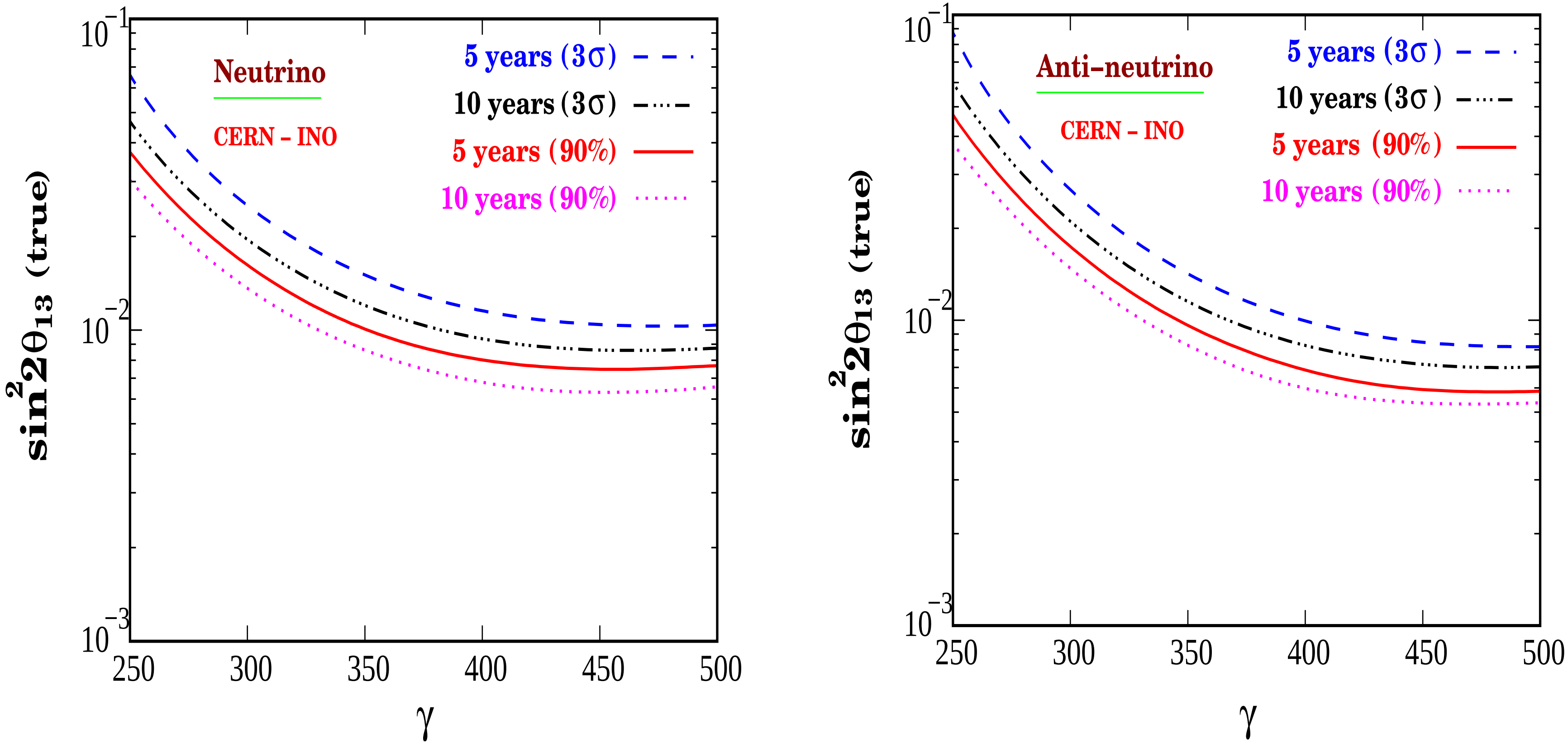
$$\chi^2(\{\omega\}) = \min_{\xi_k} \left[2\left(\tilde{N}^{th} - N^{ex} - N^{ex} \ln \frac{\tilde{N}^{th}}{N^{ex}} \right) + \sum_k \xi_k^2 \right] \; .$$

$\{\omega\}$: oscillation parameters, $\{\xi_k\}$: “pulls”, where k: runs over systematic uncertainties

$$\tilde{N}^{th}(\{\omega\}, \{\xi_k\}) = N^{th}(\{\omega\}) \left[1 + \sum_{k=1}^{K} \pi^k \xi_k \right] + \mathcal{O}(\xi_k^2) \; ,$$

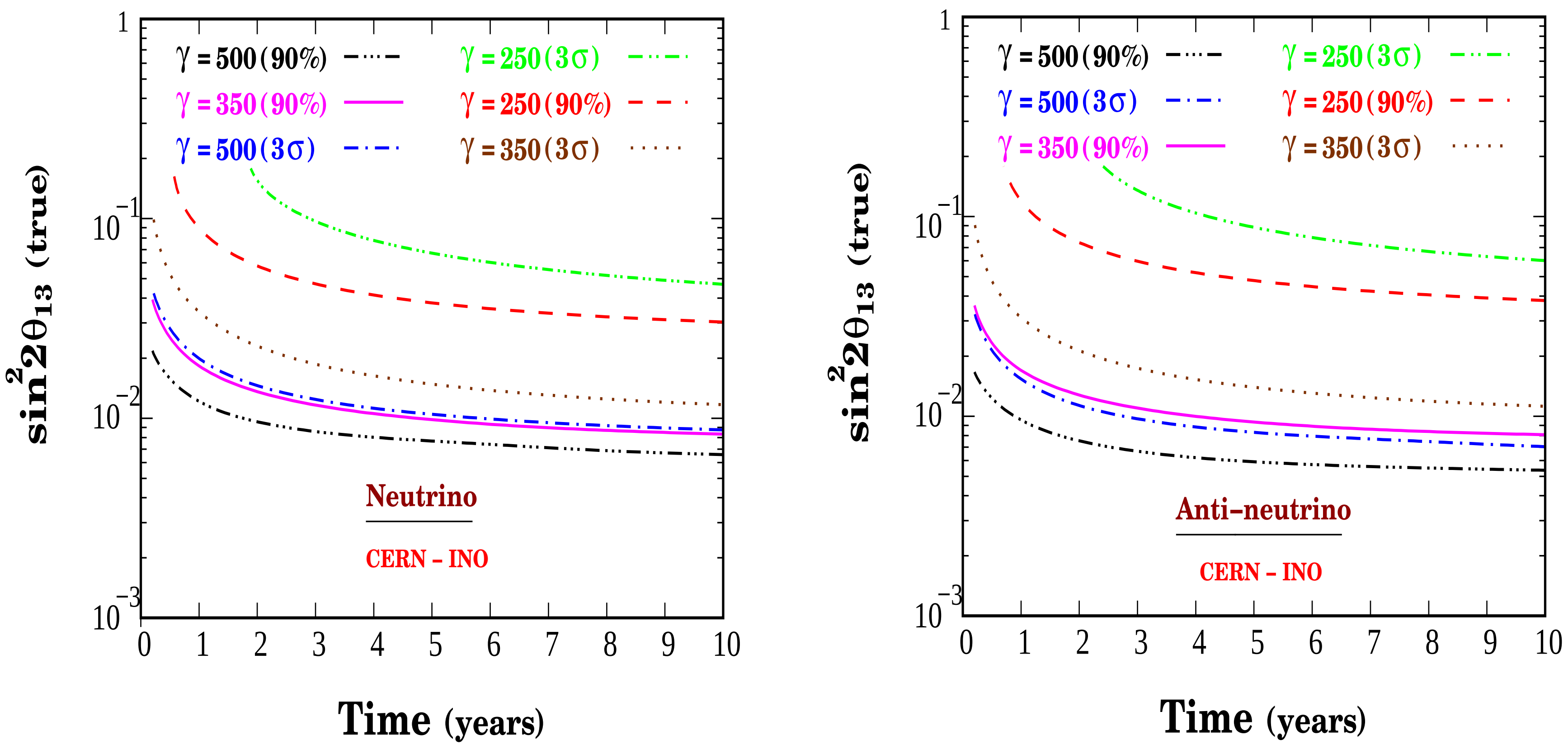
Minimise χ^2 by varying over $\{\omega\}$ and finally marginalise over Δm^2_{31}, $\sin^2 2\theta_{23}$ by minimising $\chi^2_{total} = \chi^2 + \chi^2_{prior}$

Neutrino mass ordering



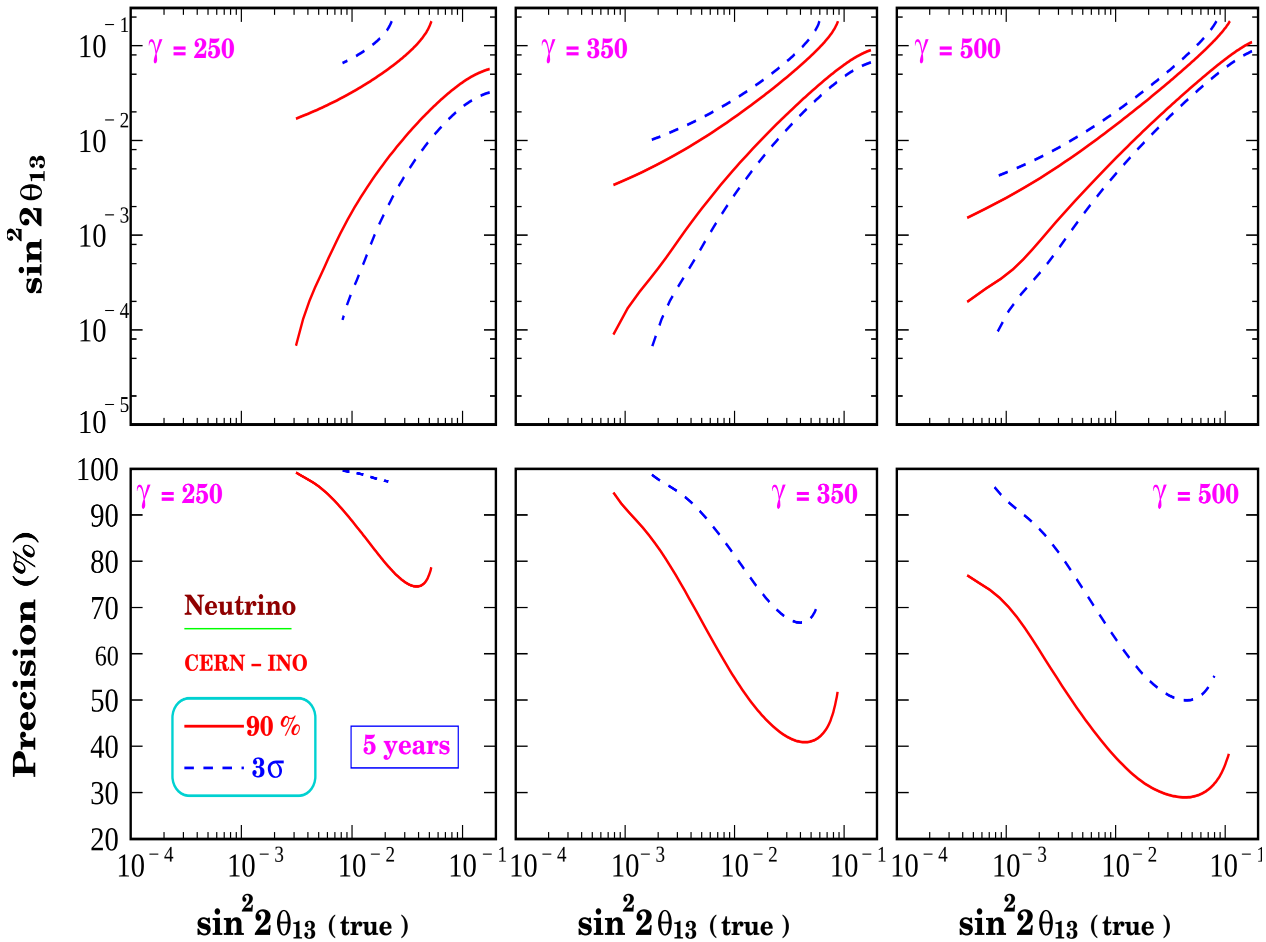
The minimum value of $\sin^2 2\theta_{13}$ as a function of the boost γ at which the wrong hierarchy can be disfavored at the 90% and 3σ C.L. For ν_e ($\bar{\nu_e}$) true hierarchy is assumed normal (inverted)

θ_{13} sensitivity



$\sin^2 2\theta_{13}$ limit below which experiment is insensitive

θ_{13} measurement



$\sin^2 2\theta$ measurement. 5 years, neutrino channel, normal hierarchy.
"measured" $\sin^2 2\theta_{13}$ (upper), corresponding precision (lower)

Conclusions

- Beta-beam source at CERN and magnetised iron calorimeter at INO: Good for exploring θ_{13} and sign(Δm^2_{23})
- The baseline is close to the “magic” value and hence avoids degeneracy problems
- Large distance captures significant matter effect
- Neutrino enenrgy for boost $\gamma \simeq 500$ gives resonant enhancement
- Results are very encouraging

Thank you!